The Quest for Lognormality in a Curved Gaussian Space-Time: An On-line Handwriting Generation Journey.

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ICDAR 2013, WASHINGTON DC, AUGUST 26.

Prelude

When I was young, I dreamed of being an astronaut, exploring space. For almost three decades now, people have been trying to make me an “internaut,” exploring the online world. Over years of people and projects, I think in fact I am only a “Cervonaut”:

Cervonaut

I enjoy
neural
stargazing

escaping
through the nebula
of spiral
ideas

traveling
at the speed
of vacuum

exploring
the white holes
of the unconscious

l'etoile

Cervonaute,
translated by
Andrea Zanin.

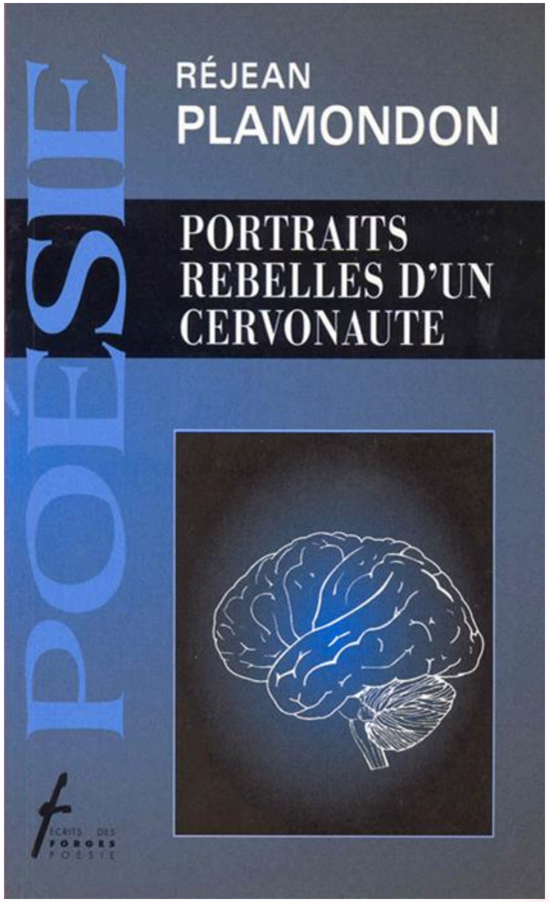
Cervonaute

J'aime faire
l'étoile
buissonnière

m'évader
par la nébuleuse
des idées
spirales

voyager
à la vitesse
du vide

en explorant
les trous blancs
de l'inconscient



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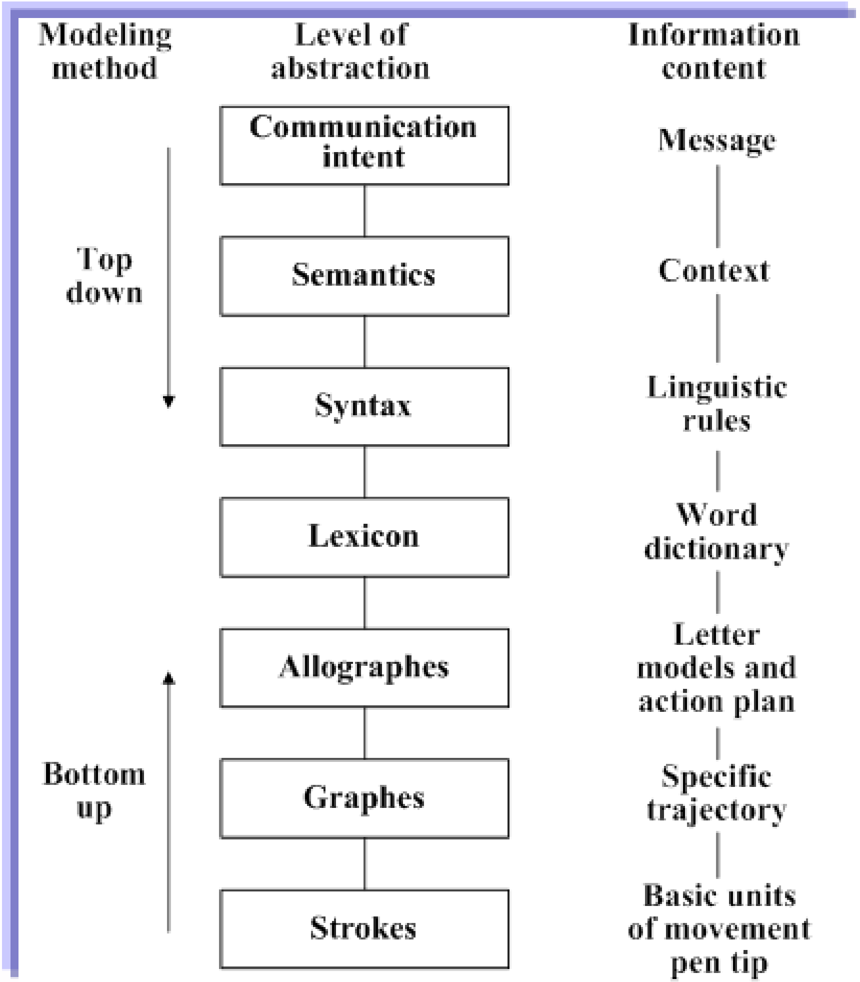
Five Opening Questions

- 1-What Is a Handwriting Stroke?
- 2-How Do We Handwrite?
- 3-Can We Recover the Action Plans?
- 4-Is the Theory Physiologically Meaningful?
- 5-Where Do We Go From Here?

1 - What is a Handwriting Stroke?

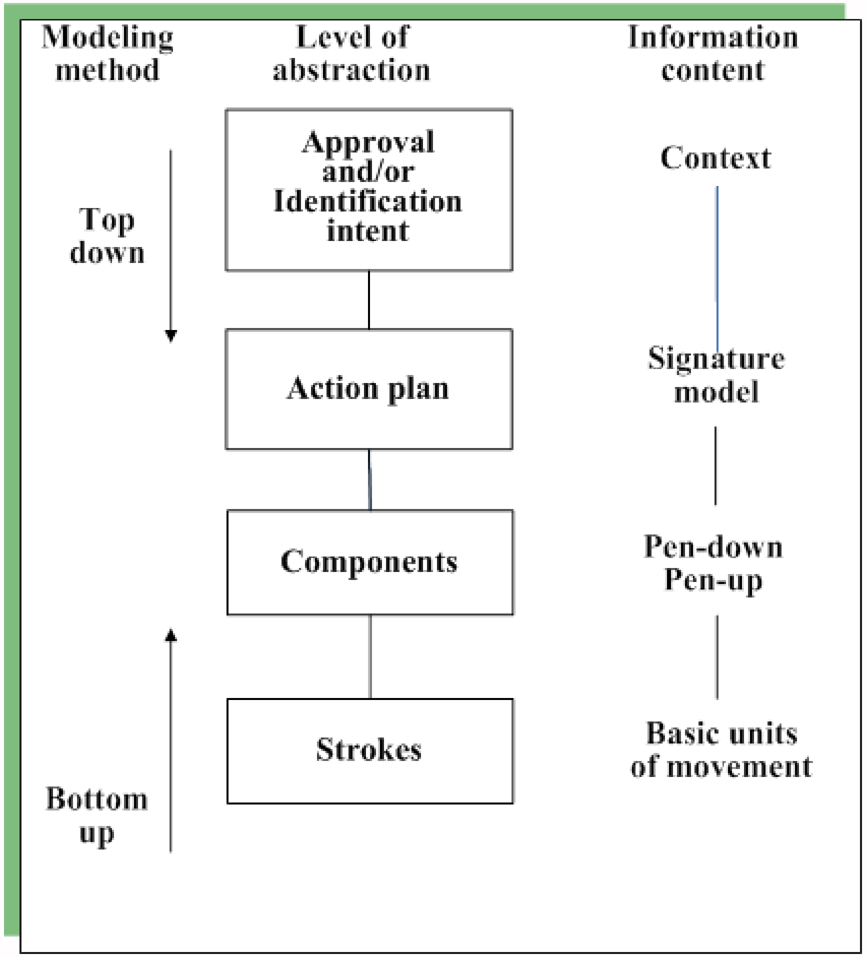
Searching for the fundamental unit of Handwriting

Handwriting Generation



PLAMONDON, R., LOPRESTI, D., SCHOMAKER, L.R.B., SRIHARI, R., “On-Line Handwriting Recognition”, Encyclopedia of Electrical and Electronics Engineering, J.G. Webster (Ed.), John Wiley & Sons, N.Y., vol. 15, 1999, p. 123-146.

Signature Generation



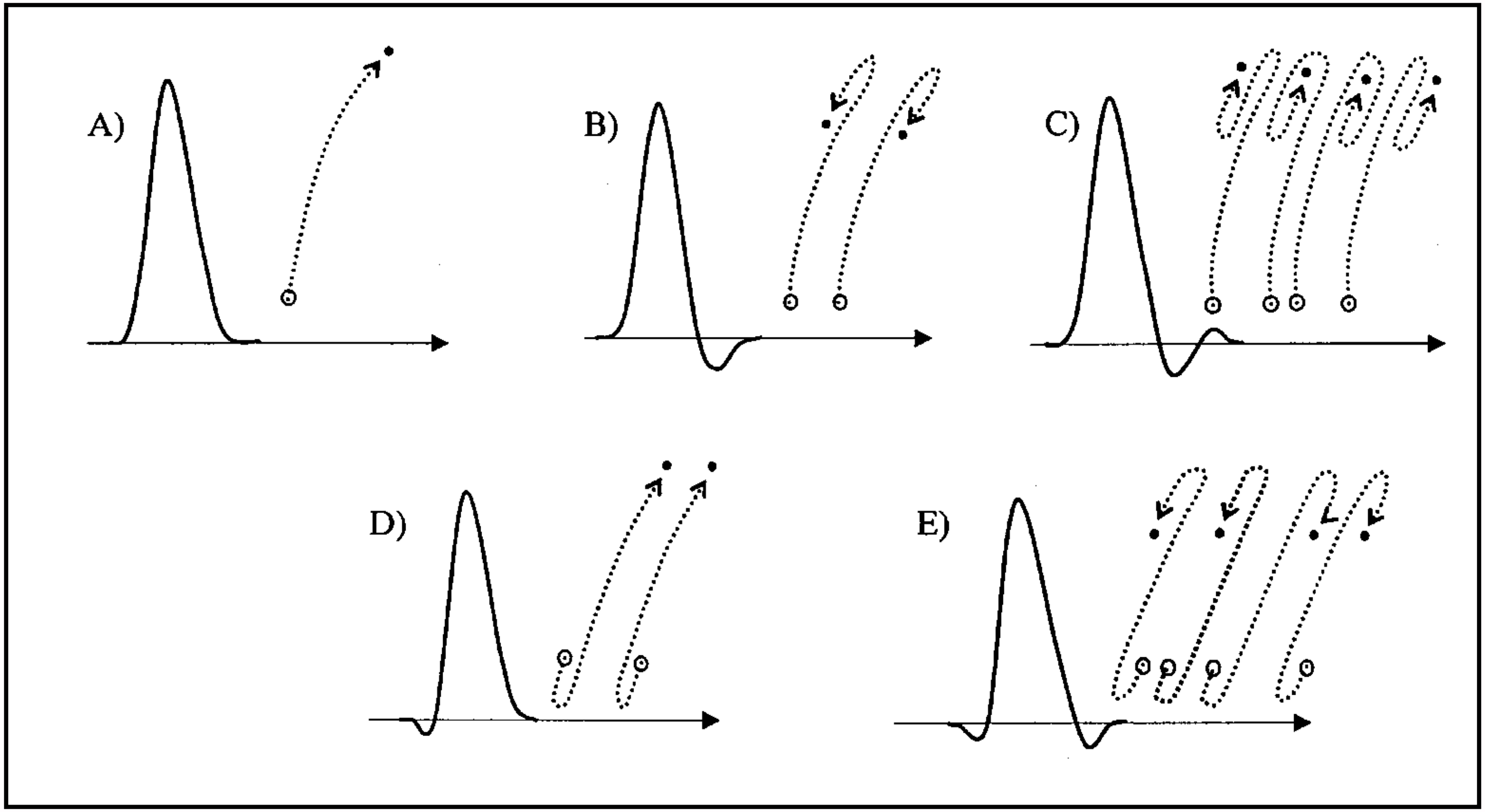
PLAMONDON, R., PIRLO, G., IMPEDOVO, D.," On-line Signature Verification" in Handbook of Document Image Processing and Recognition, D.Doermann , K.Tombre Eds, In Press, Springer, 2012.

Model Classification

- Criterion: Motor Control Basic Hypothesis

  - Equilibrium Point (Feldman ,Bizzi, Hollerbach…)
  - Neural Networks (Bullock, Schomaker, Gangadhar…)
  - Optimization Principles (Flash, Hogan, Kawato…)
  - Non-linear Dynamics (Kelso, Athenes, Zazone…)
  - Proportionality and Convergence (Plamondon)

Typical Velocity Profiles and Trajectories



WOCH, A., PLAMONDON, R., «Using the Framework of the Kinematic Theory for the Definition of a Movement Primitive», Motor Control, Special Issue, vol. 8, No 4, pp. 547-557, 2004.

Basic Characteristics of a Single Stroke

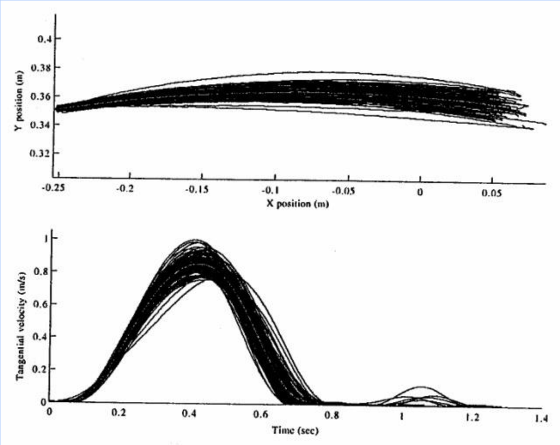


Figure : Typical trajectories and velocity profiles

- **No visual feedback**
- **Speed accuracy trade-offs**
- **Almost rectilinear trajectory of the end effector**
- **Asymetric bell-shaped velocity profile**
- **Up to two secondary velocity peaks**
- **Possible direction inversion at the begining and/or at the end**

PLAMONDON, R., ALIMI, A., «Speed/Accuracy Tradeoffs in Target Directed Movements», Behavioral and Brain Sciences, vol. 20, No 2, pp. 279-349, 1997.

Fundamental Hypothesis

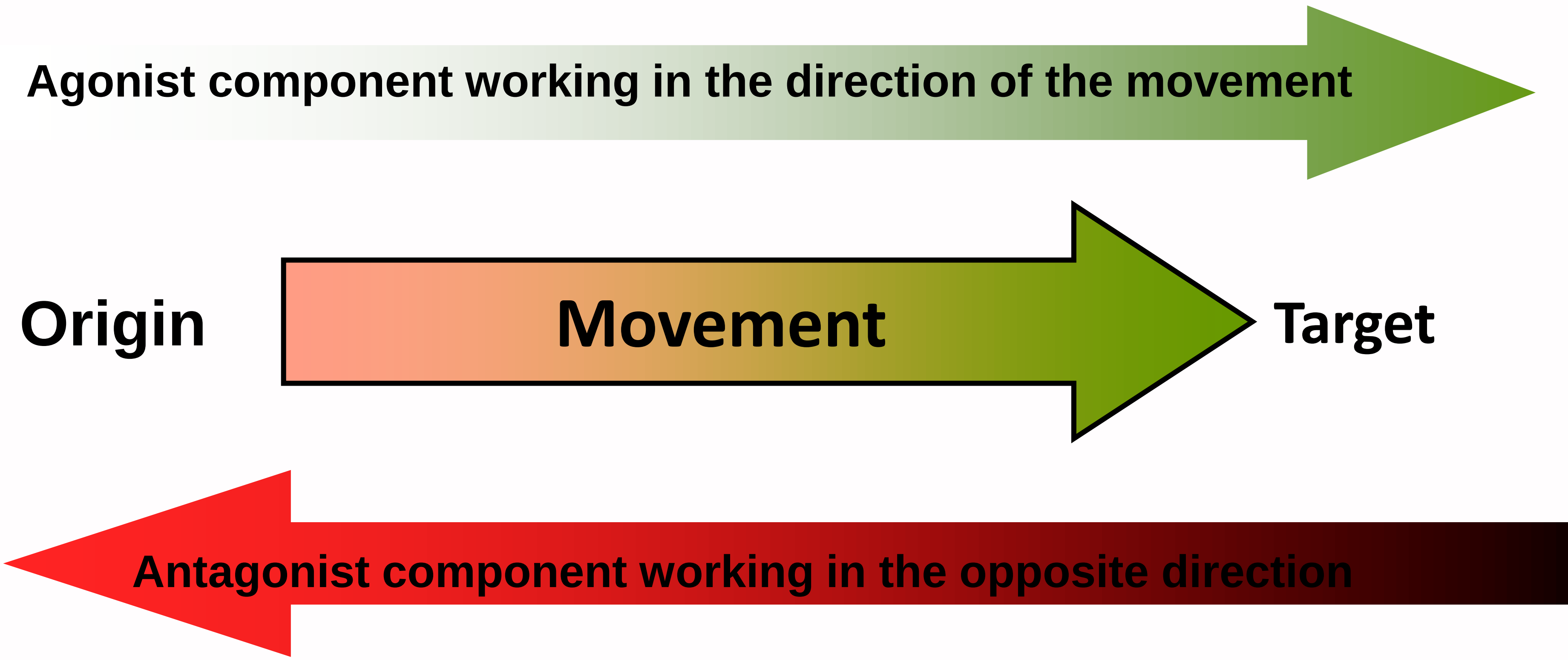
The invariant properties
of these single strokes reflect
the asymptotic behavior of complex systems,
made up of a large number of coupled
neuromuscular networks.

→ **EMERGENCE FROM**
ASYMPTOTIC CONVERGENCE

Basic Tool

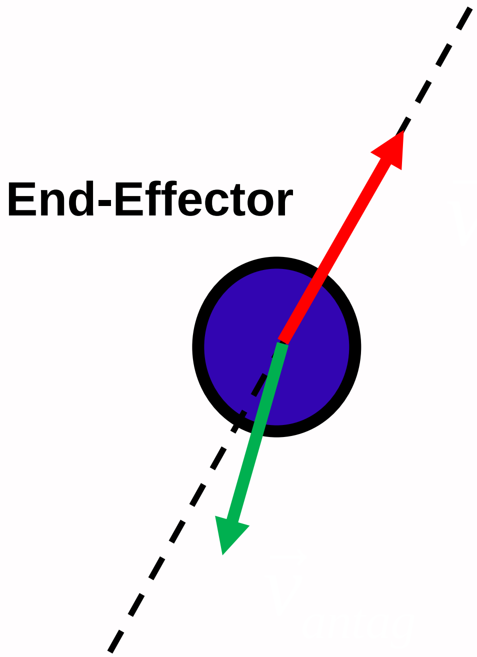
The Central Limit Theorem can be used to point out emergent phenomena in these complex systems.

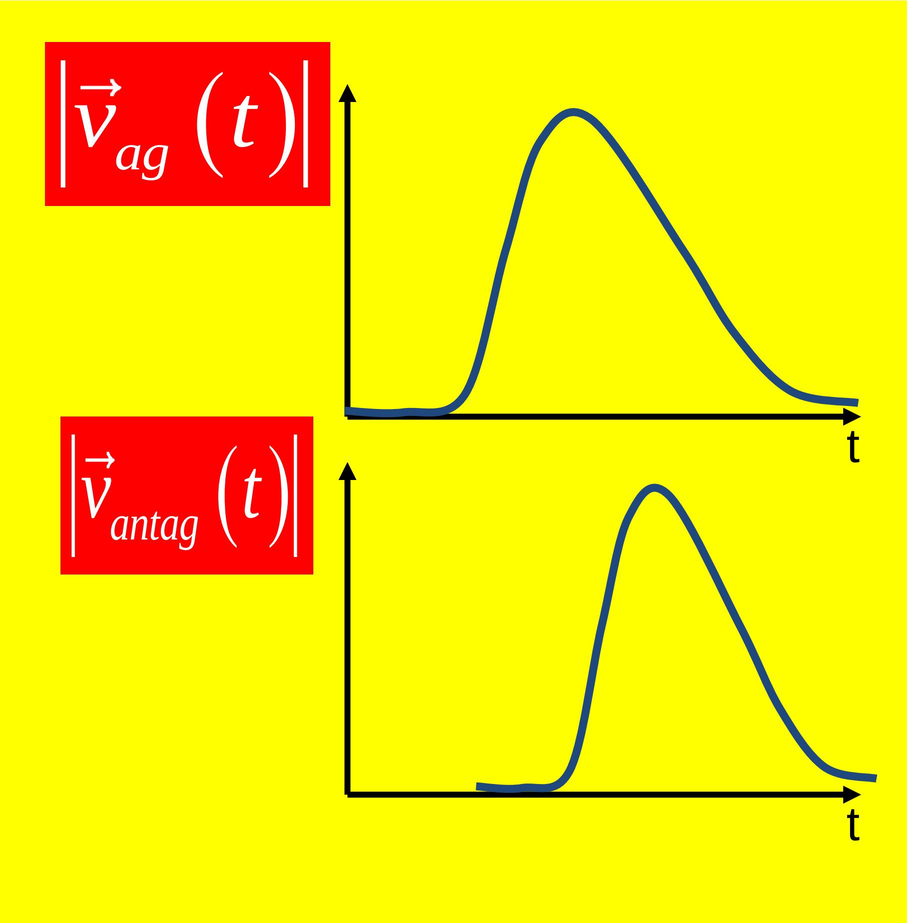
Agonist – Antagonist Synergy



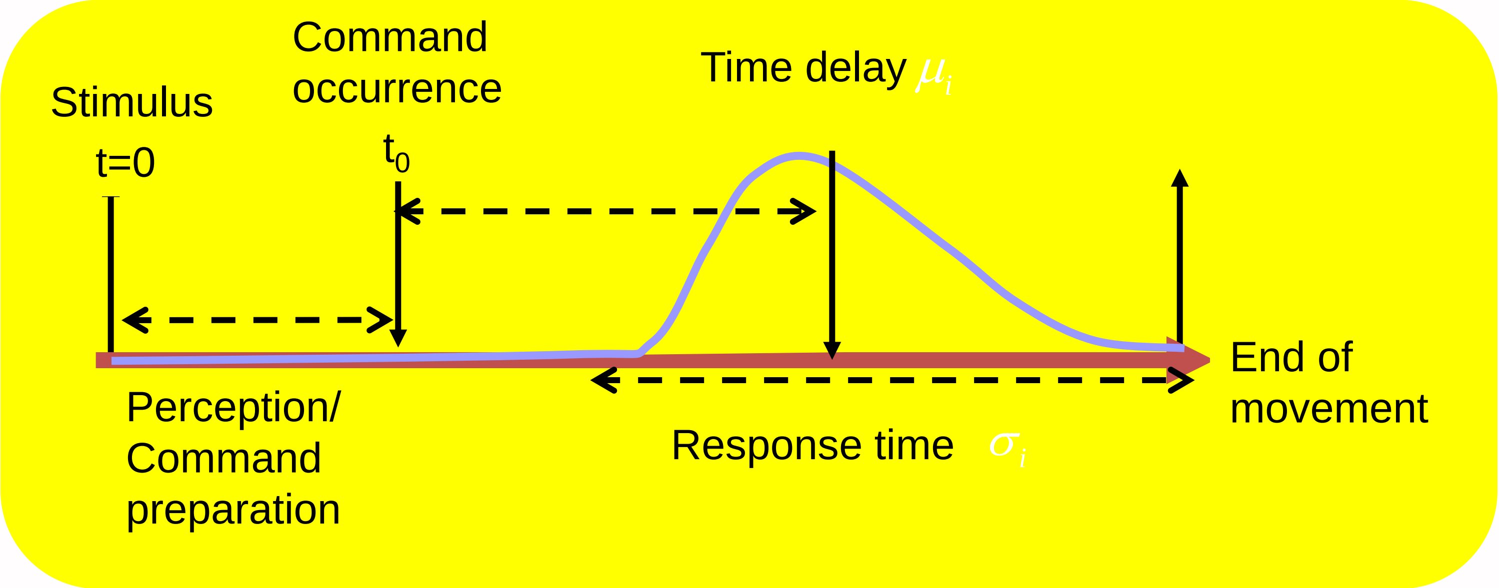
Kinematic Theory : spatial analysis

Vectorial summation

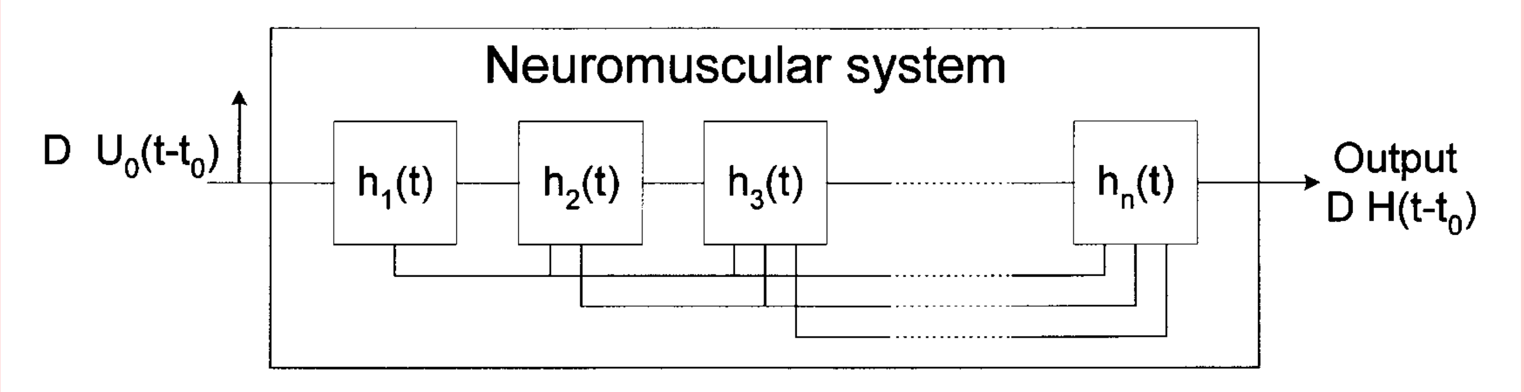




Temporal analysis of a movement component (agonist or antagonist)



- Mathematical proof based on the Central Limit Theorem
- Convergence of the NMS impulse response towards a lognormal profile



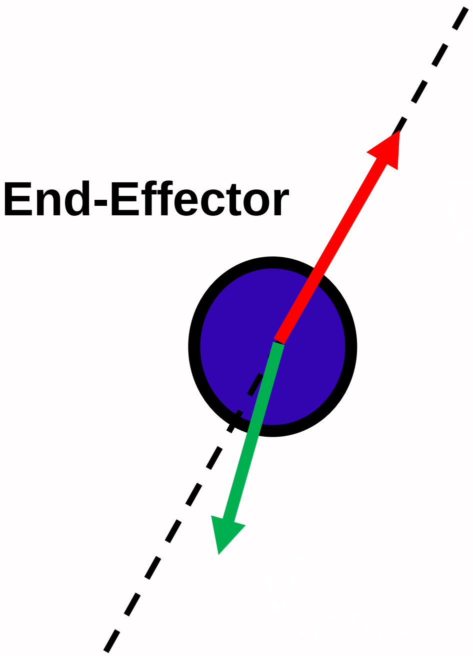
- Hypothesis

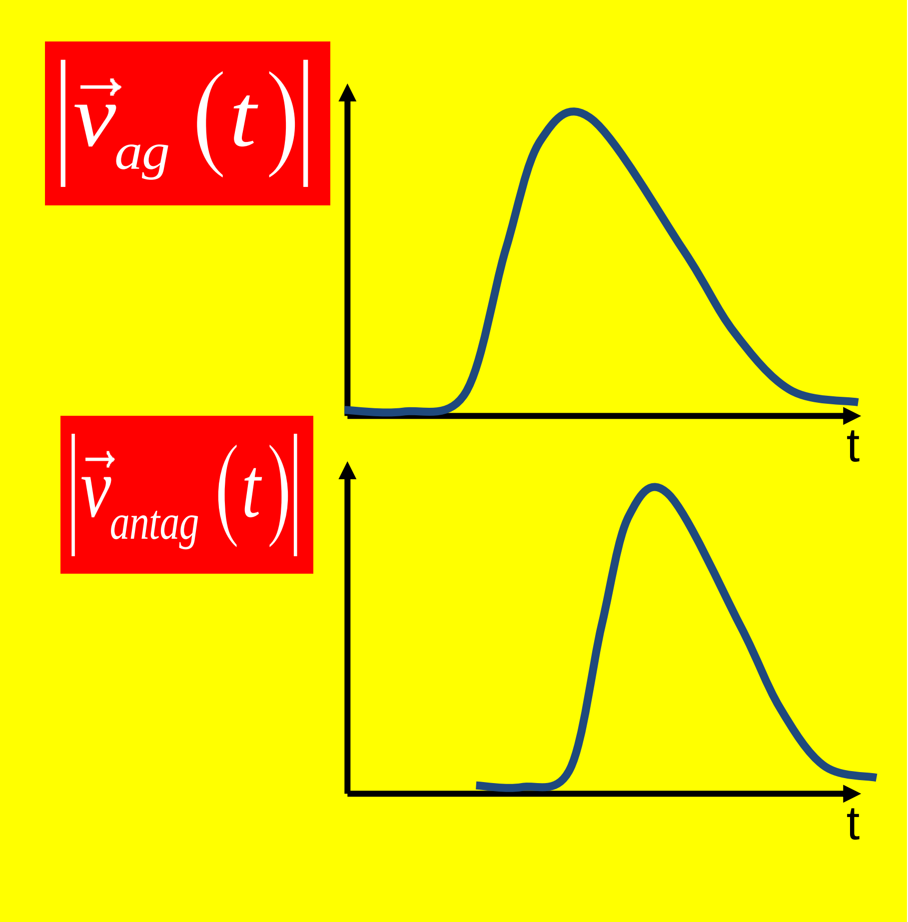
$$T_n = (1+\varepsilon_n)T_{n-1}$$

$$n \rightarrow \infty$$

$$H(t-t_0) \Rightarrow \Lambda(t; t_0, \mu, \sigma^2)$$

Vectorial summation





Lognormal Velocity

$$\left|\vec{v}(t,P)\right| = D\Lambda(t;t_o,\mu,\sigma)$$

$$= \frac{D}{\sigma\sqrt{2\pi}(t-t_0)}\exp-\left[\frac{\left[\ln(t-t_0)-\mu\right]^2}{2\sigma^2}\right]$$

Velocity profile of a single stroke: Sigma-Lognormal Model

$$\vec{v}(t) = \vec{v}_{ag}(t) + \vec{v}_{antag}(t)$$

Special case: perfect opposition of the agonist and the antagonist components

$$v(t) = v_{ag}(t) - v_{antag}(t)$$

Delta-Lognormal Model

A BRIEF PAUSE

**A STROKE IS THE IDEAL OUTPUT
OF A NEUROMUSCULAR SYSTEM
REFLECTING ITS IMPULSE RESPONSE
WHICH RESULTS
FROM AN EMERGING BEHAVIOR
PREDICTED
BY THE CENTRAL LIMIT THEOREM**

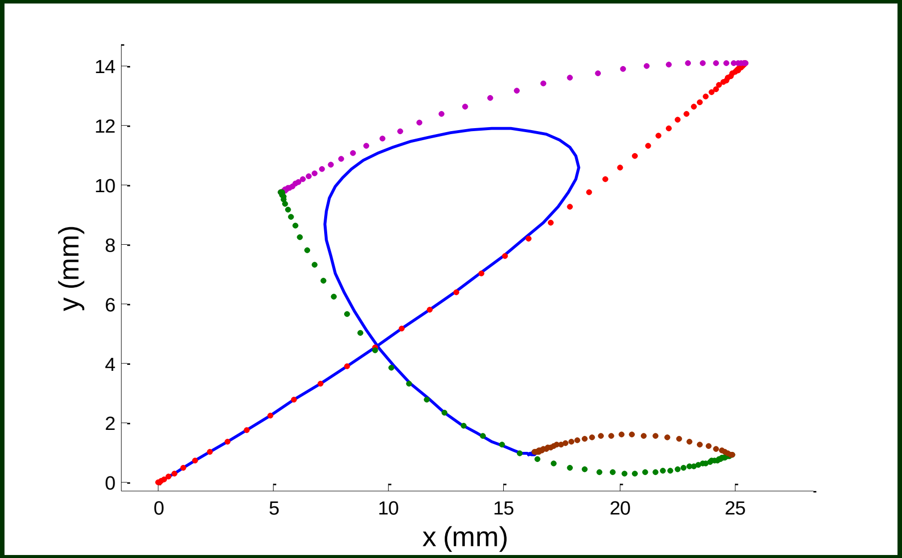
2 - How do we Handwrite ?

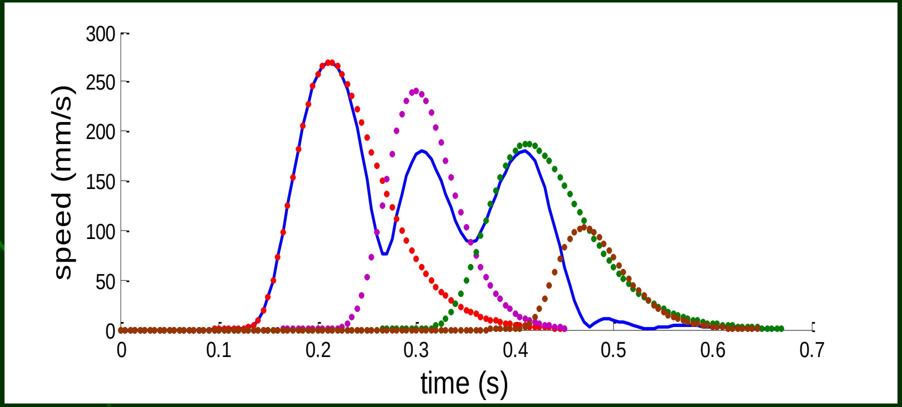
By taking advantage of the intrinsic properties of strokes

Sigma-Lognormal model

- Discontinuous action plan
- Virtual targets
- Vectorial summation of curved strokes
- Time overlap
- Individual strokes hidden in the signal
  - Velocity profiles : Lognormal functions
  - Direction angle profiles: Error functions (Erf)

Complex movement generation





Component representation and synergy

- A neuromotor component is acting around a pivot point.
- Direction of the i[th] component trajectory :

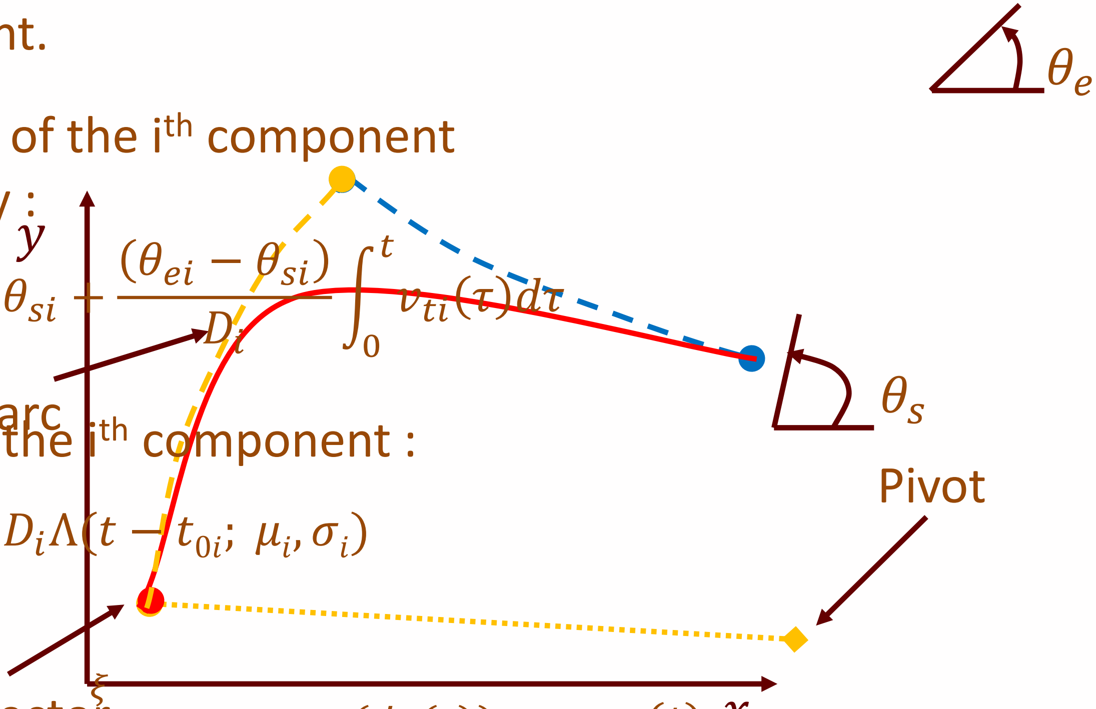
$$\varphi_i(t) = \theta_{si} + \frac{(\theta_{ei} - \theta_{si})}{D_i} \int_0^t v_{ti}(\tau)d\tau$$

- Speed of the i[th] component :

$$v_{ti}(t) = D_i \Lambda(t - t_{0i};\ \mu_i, \sigma_i)$$

- Synergy :

$$v(t) = \sum_{i=1}^{\xi} v_{ti}(t) \begin{bmatrix} \cos(\phi_i(t)) \\ \sin(\phi_i(t)) \end{bmatrix} = \begin{bmatrix} v_x(t) \\ v_y(t) \end{bmatrix}$$



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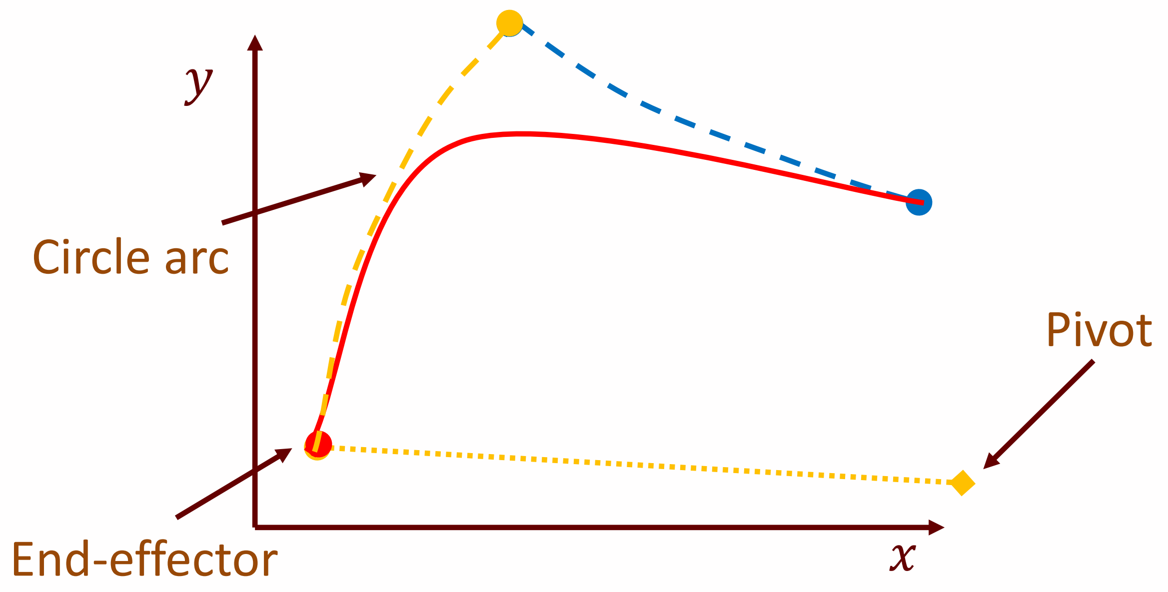

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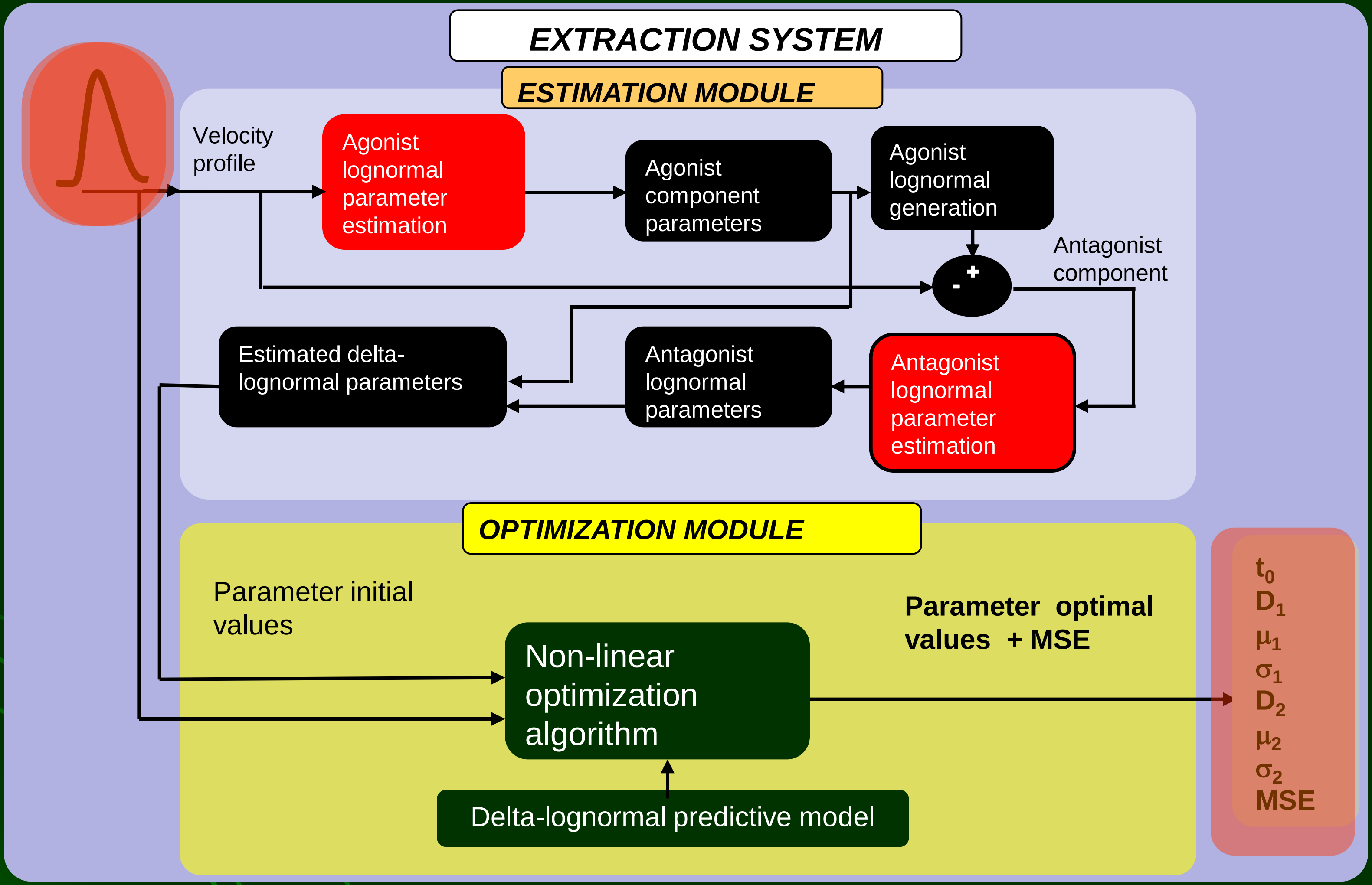
3 - Can we recover the action plans ?

A reverse engineering problem: Lognormal parameter extraction

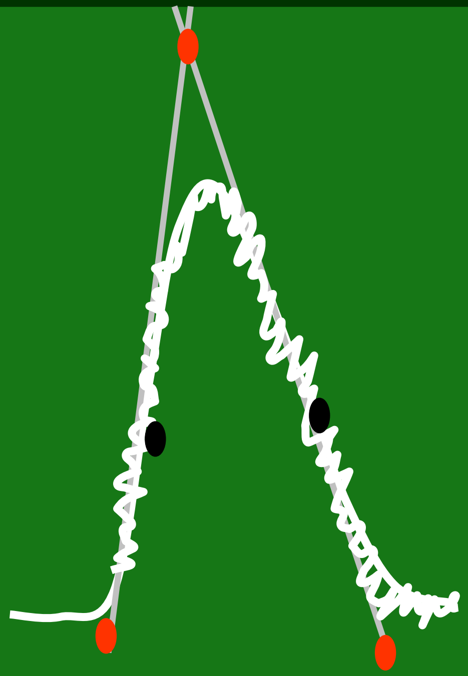
Kinematic Theory : Extraction system : Architecture



estimation algorithm #1: INFLEX

INFLEX Algorithm

Graphical method
based on three points defined
by the intersection
of the inflexion point tangents
and a table of values
computed by Wise (1963)



Limitations

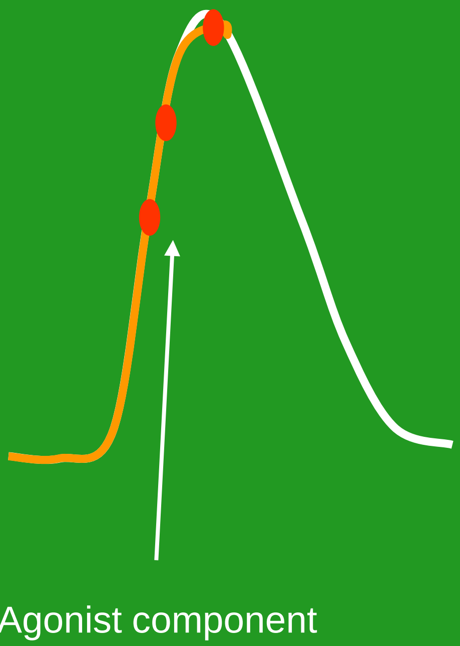
- negative t_0
- very sensitive to noise
- sensitive to the antagonist vs agonist component location

GUERFALI, W., PLAMONDON, R., Signal Processing for the Parameter Extraction of the Delta Lognormal Model (ΔΛ),
In C. Archibald and P. Kwok (Ed.)**, Research in Computer and Robot Vision, World Scientific, S**ingapore, 1*995, pp. 217-232.*

estimation algorithm #2: MINIT

MINIT Algorithm

Use of three points :
the maximum and
two arbitrary points
taken on the rising phase
of the velocity signal



Advantages

Less sensitive to noise

Limitations

Sensitive to the profile asymmetry

Sensitive to the antagonist vs. agonist component location

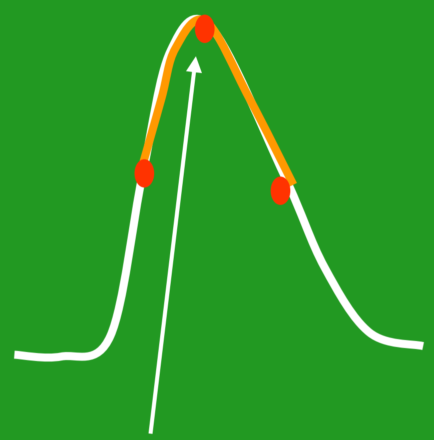
PLAMONDON, R., LI, X., DJIOUA, M., Extraction of Delta lognormal parameters from Handwriting Strokes, **Frontiers of Computer Science in China**, vol. 1, No1, pp.106-113, 2007.

estimation algorithm #3: Xzero

XZERO Algorithm

Use of three points :
the maximum and
the two inflexion points

Exploit the analytical relationships
between these points
and the four parameters
of a single lognormal



Agonist Component

Advantages

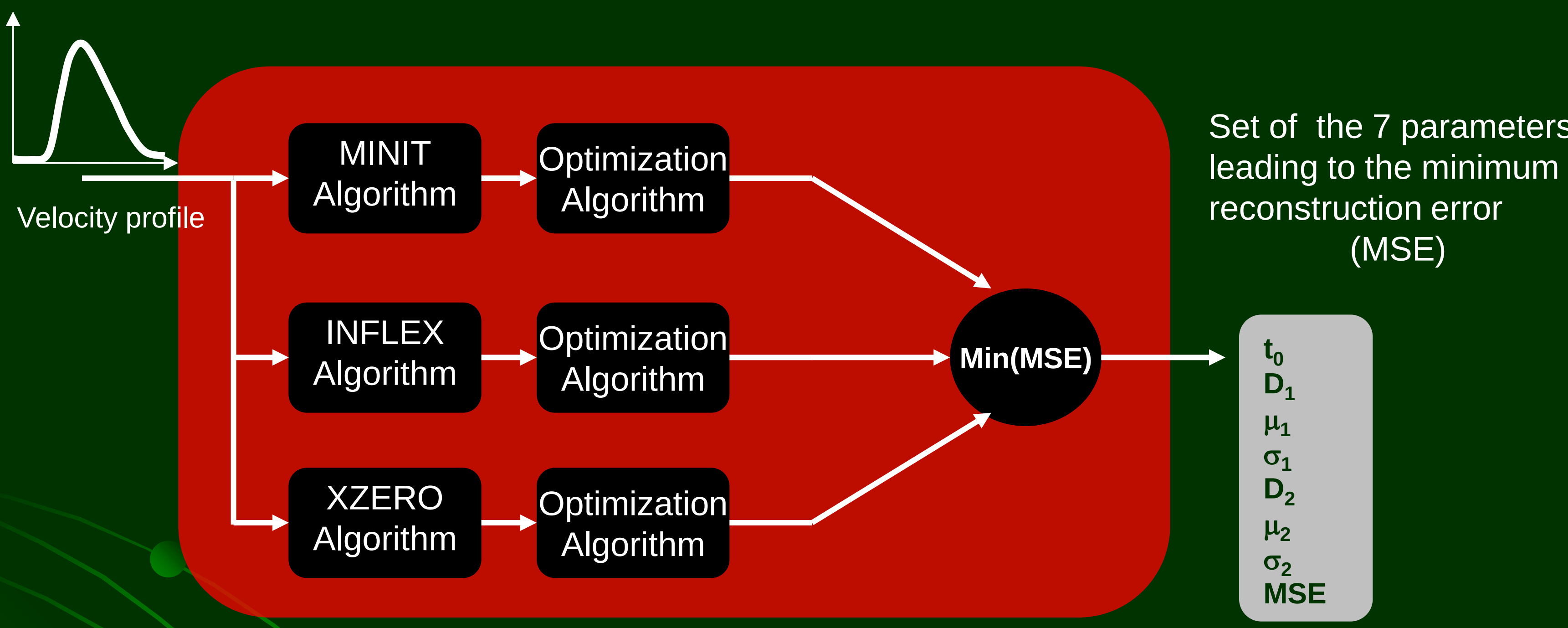
- Less sensitive to noise
- Independent of the antagonist vs. agonist position

Limitations

Sensitive to the number of velocity samples between the two inflexion points

DJIOUA, M., PLAMONDON, R., “A New Algorithm and System for the Characterization of Handwriting Strokes with Delta-Lognormal Parameter” **IEEE Transactions on Pattern Analysis and Machine Intelligence**, vol. 31, No 11, November 2009, pp. 2060-2072.

Extraction system architecture



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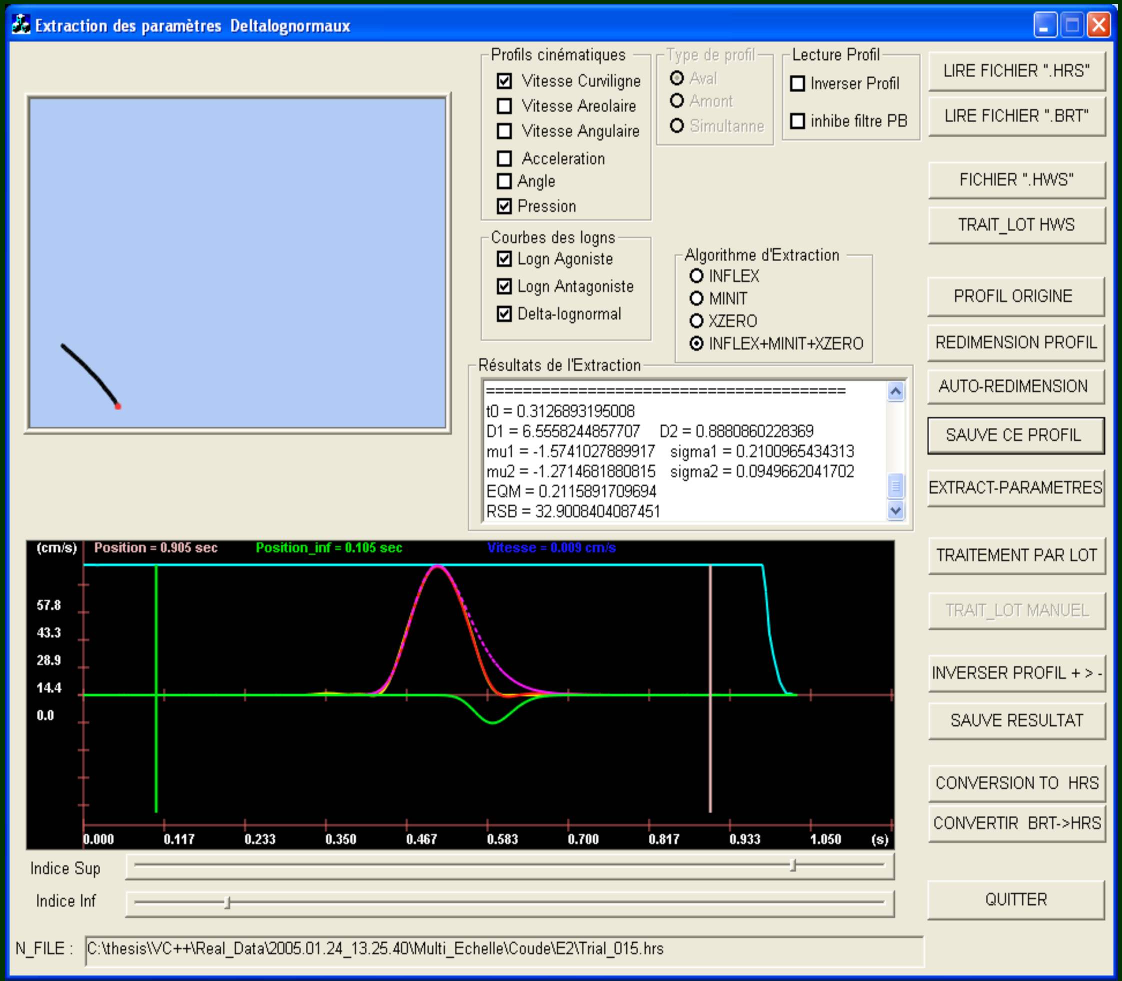
Delta-lognormal Extraction System

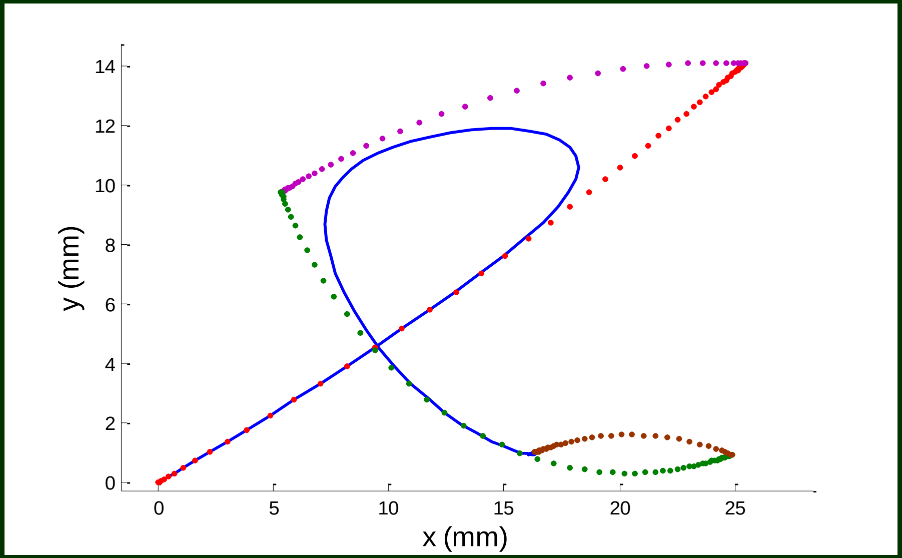
Implementation of the three extraction algorithms

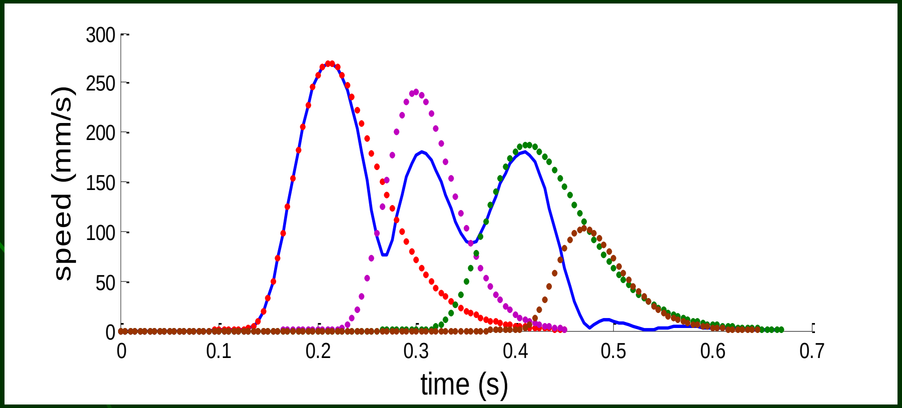
+

Extraction system characterization



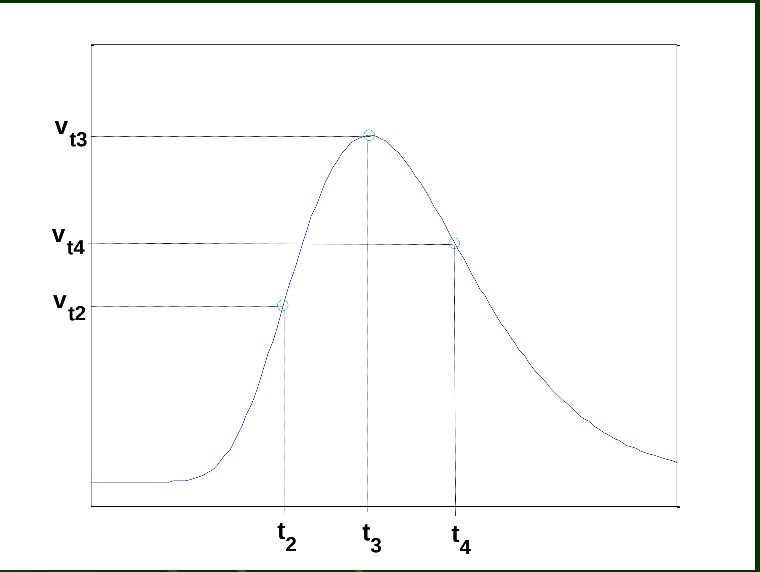
Complex movement generation



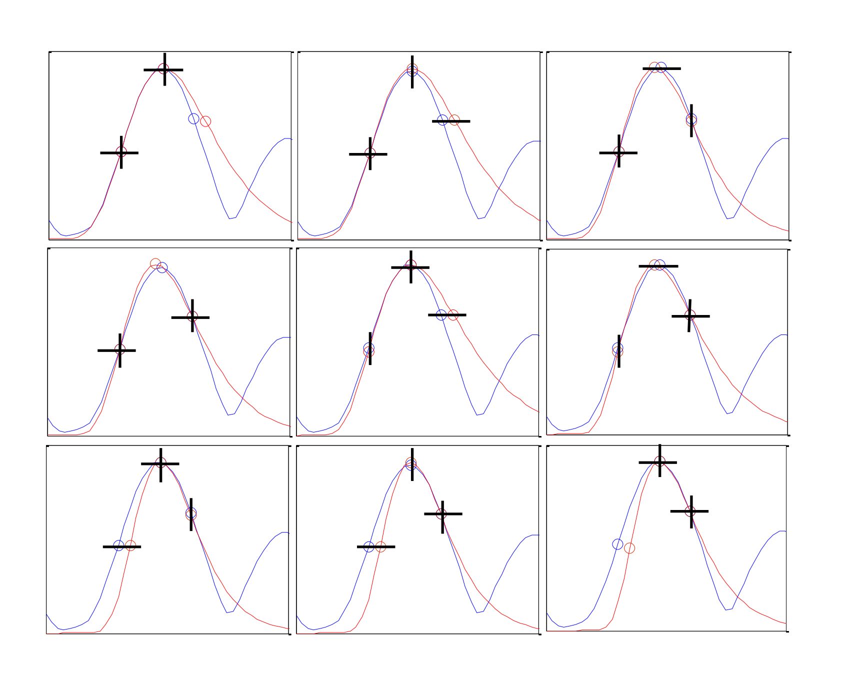


XZERO Robust Algorithm

Original XZERO

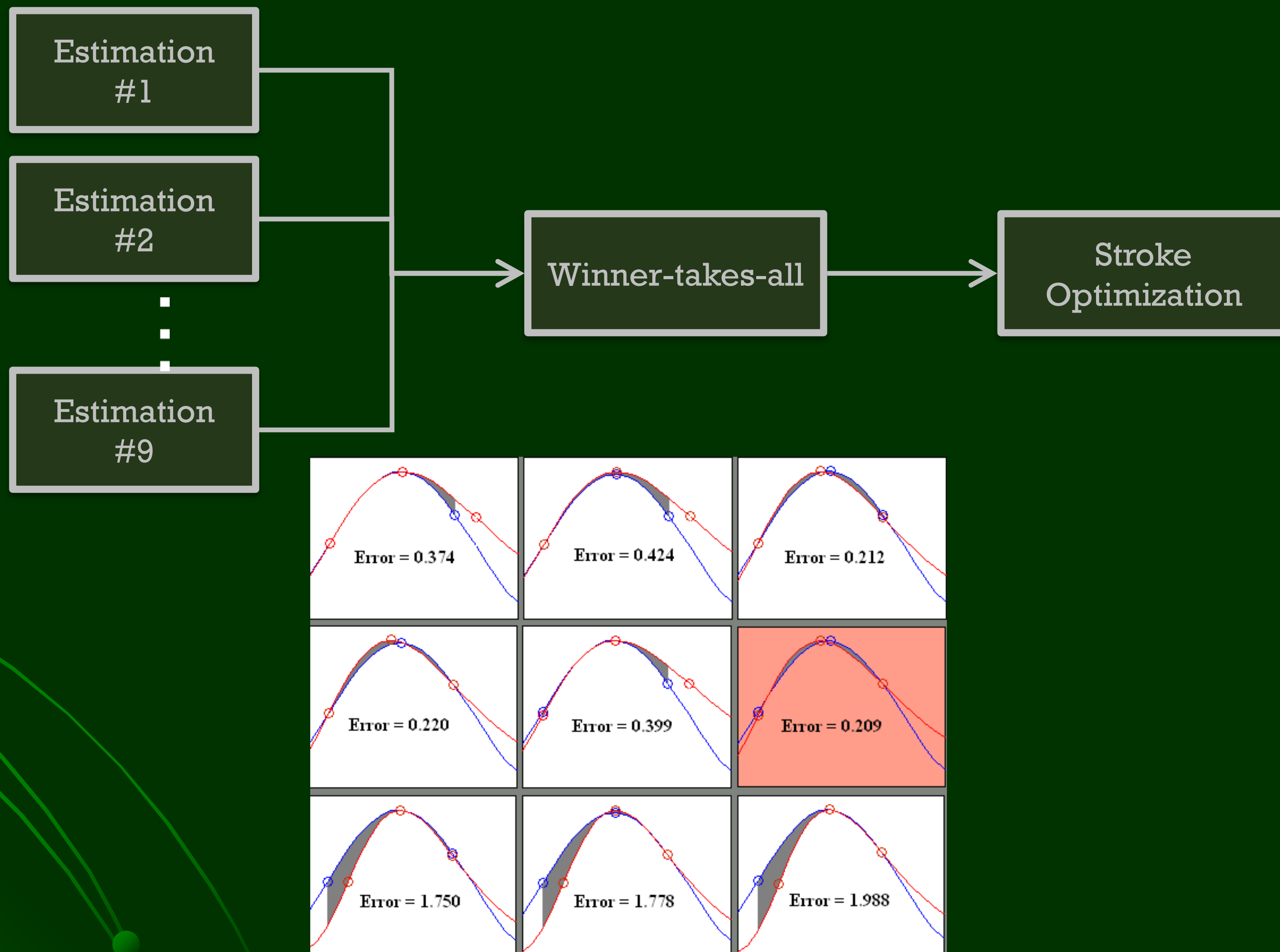


9 Analytic relationships for the initial estimation of the parameters

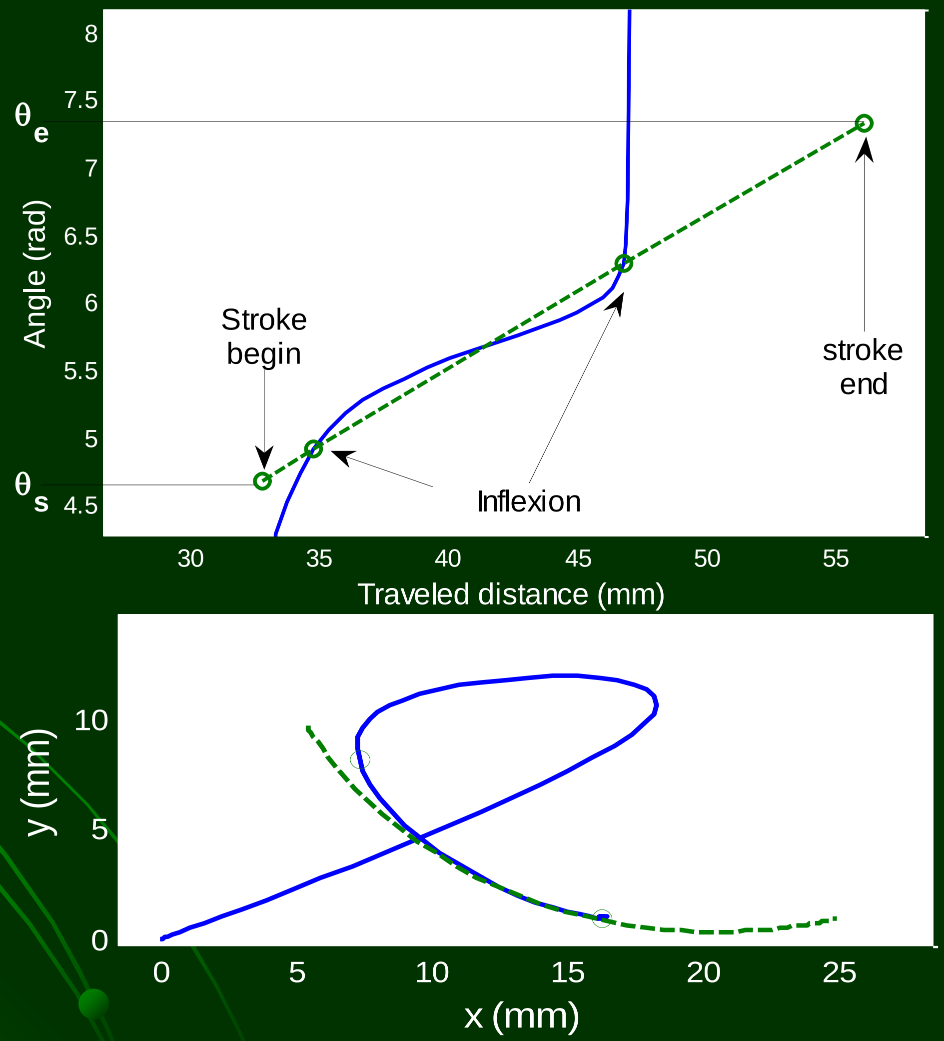


O'REILLY, C., PLAMONDON, R., Development of a Sigma-Lognormal Representation for On-Line Signatures. **Pattern Recognition**, Special Issue on Frontiers in Handwriting Recognition, vol.42, 2009, pp. 3324-3337.

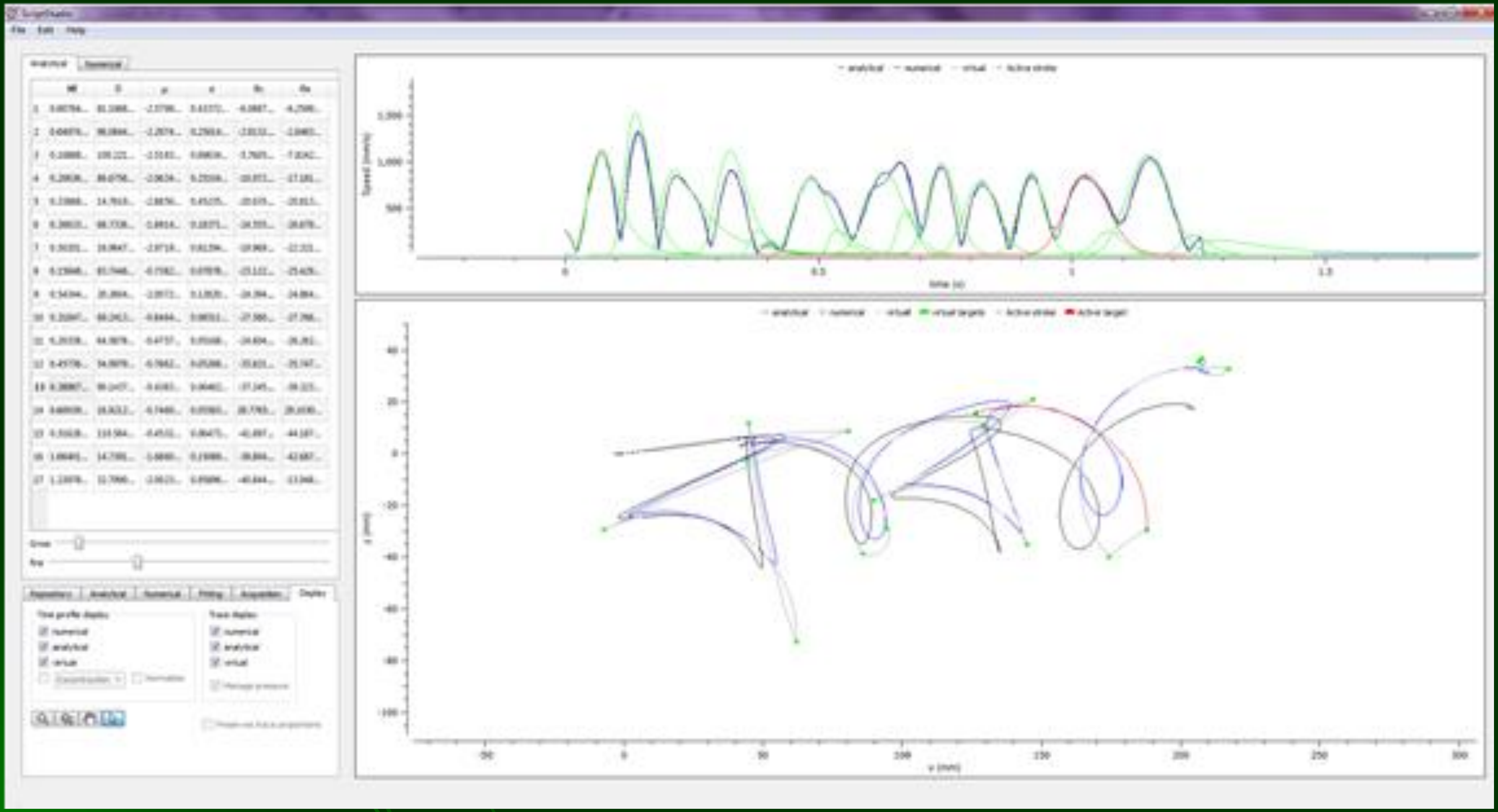
Sigma lognormal Parameter Estimation



Angle Estimation



SCRIPTSTUDIO DEMO



Fitting — Four parameter extractors

1 INFLEX-INITRI-XZERO (IIX)

- ΔΛ representation (locally optimal)
- Fast reaching motion

M. Djioua and R. Plamondon, "A new algorithm and system for the characterization of handwriting strokes with delta-lognormal parameters," *IEEE Trans Pattern Anal Mach Intell,* vol. 31, pp. 2060-72, Nov 2009.

2 Branch and bound (B&B)

- ΔΛ representation (globally optimal)
- Fast reaching motion

C. O'Reilly and R. Plamondon, "A globally optimal estimator for the Delta-Lognormal modeling of fast reaching movements " *IEEE Trans. on System, Man and Cybernetics. Part B. Cybernetics,* in press.

3 Robust X_0

- ΣΔ representation
- Complex and arbitrary movements

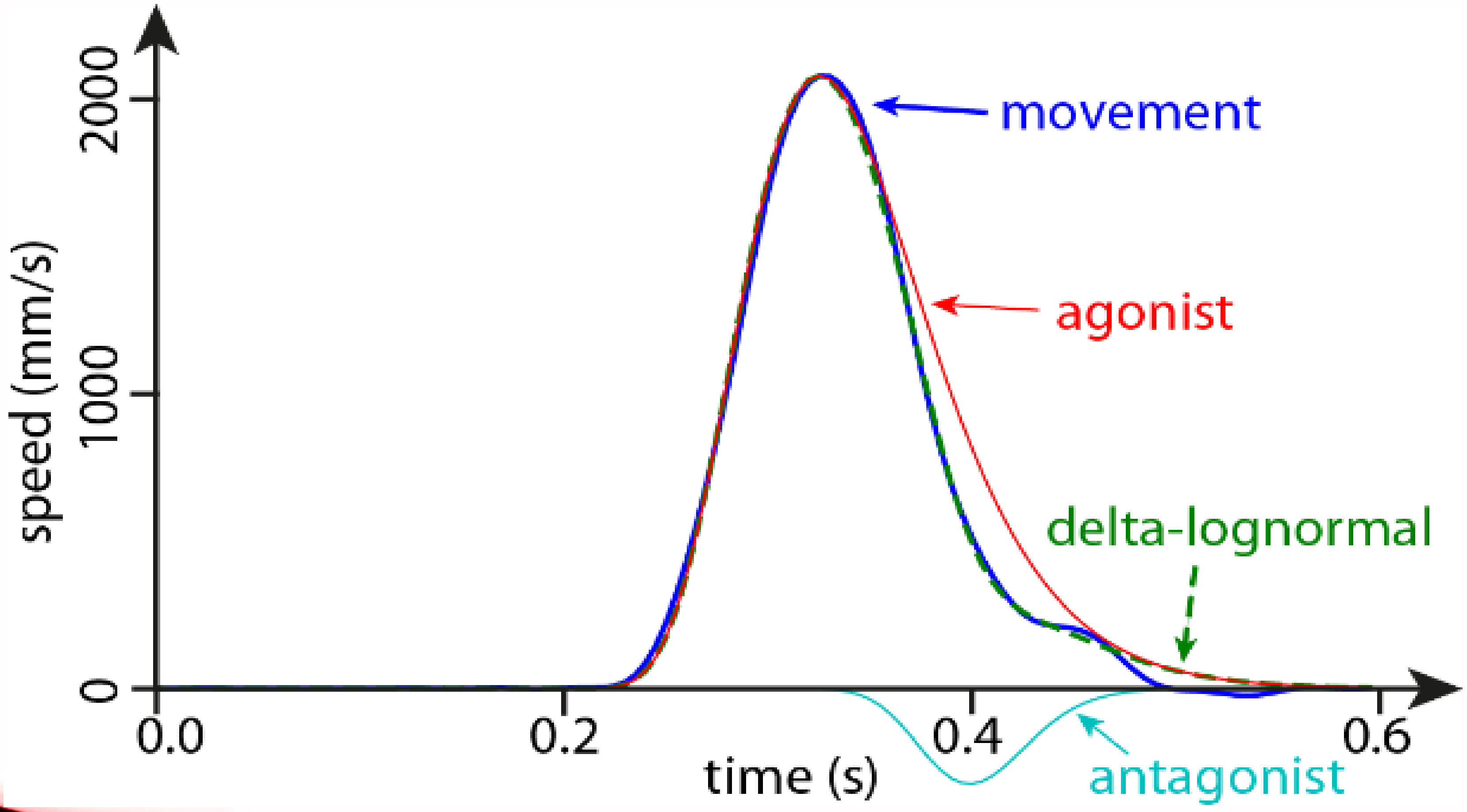
C. O'Reilly and R. Plamondon, "Development of a Sigma-Lognormal representation for on-line signatures," *Pattern Recognition,* vol. 42, pp. 3324-3337, 2009.

4 Prototype based

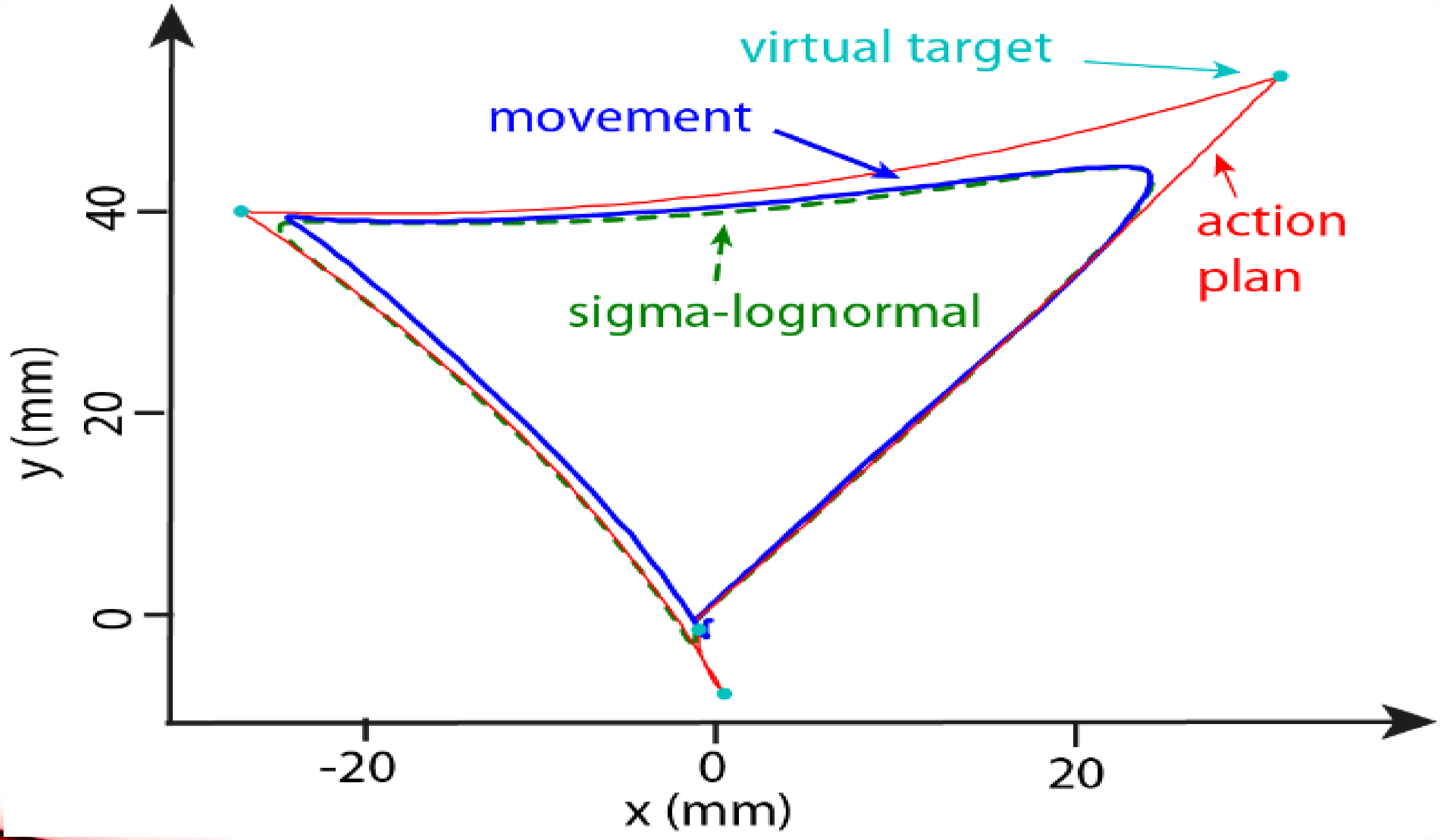
- ΣΔ representation
- Complex and stereotypical movements
- Allow performing ANOVA of the ΣΔ parameters

O'Reilly and R. Plamondon, "Prototype-based methodology for the statistical analysis of local features in stereotypical handwriting tasks," proceedings of 20th Int. Conference on Pattern Recognition , Istanbul, Turkey, pp. 1864-1867, 2010.

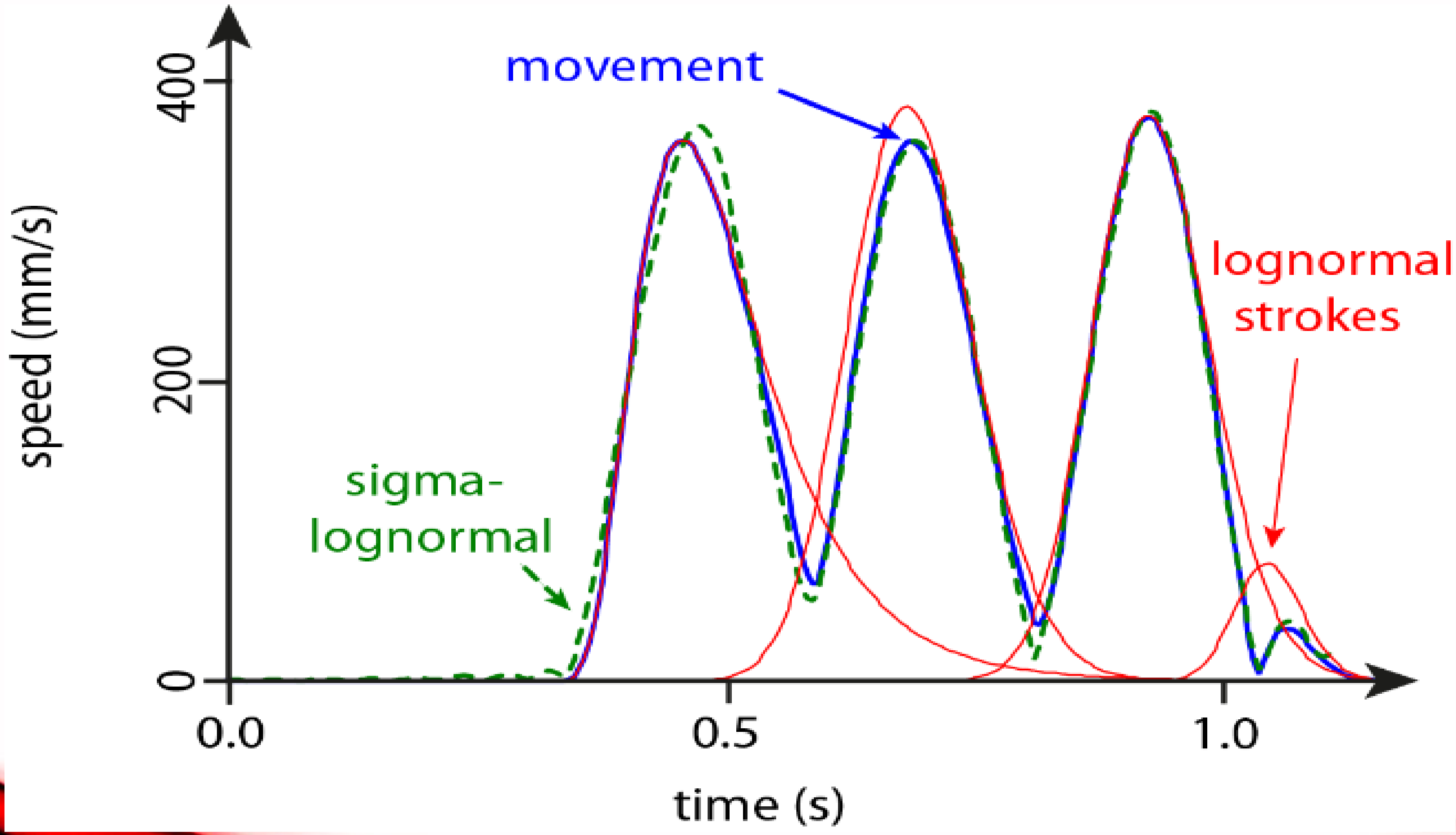
Delta-lognormal model



Sigma-lognormal model



Sigma-lognormal model



4 - Is the theory Physiologically meaningfull ?

Testing the basic underlying hypotheses

Theoretical Problem No 1

Experimental investigation of t_0

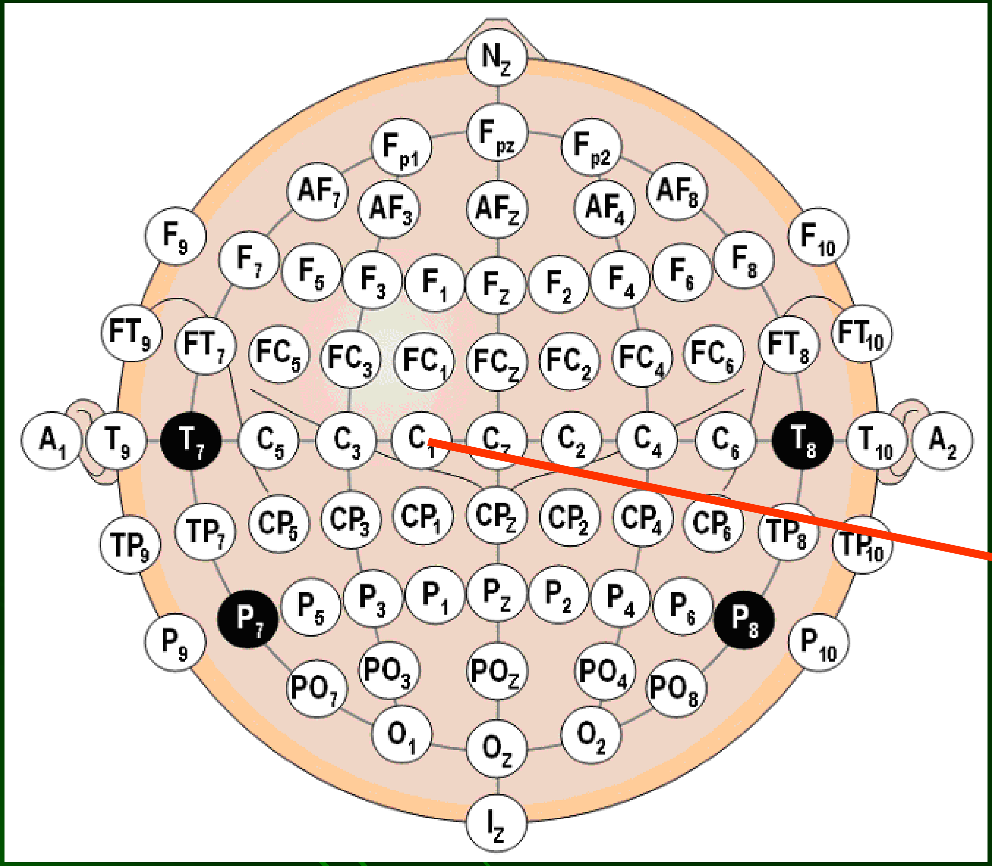
O'REILLY, C., PLAMONDON, R., LANDOU M. K., STEMMERS B.,. Prediction of the Time Occurrence a Motor ERP based on the Kinematic Analysis of Movement with the Delta-Lognormal Model. European Journal of Neuroscience, Vol. 37, pp. 173–180, 2013.

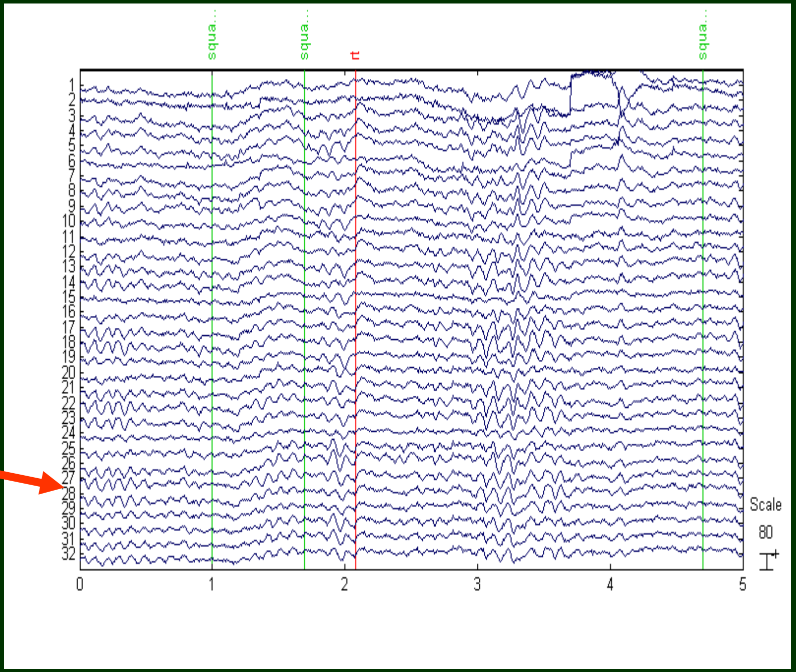
Experimental Protocol

- Recording evoked potentials with 64 electrodes

Typical trial

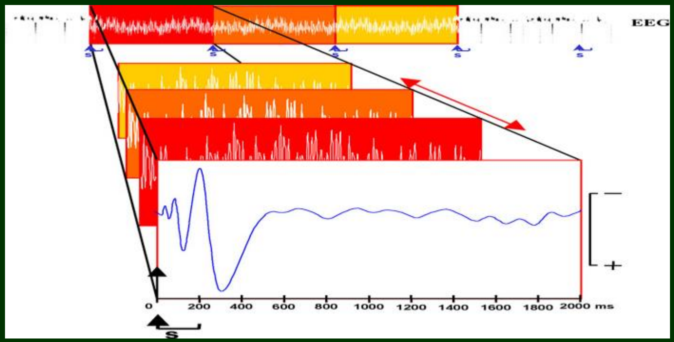
EEG recording

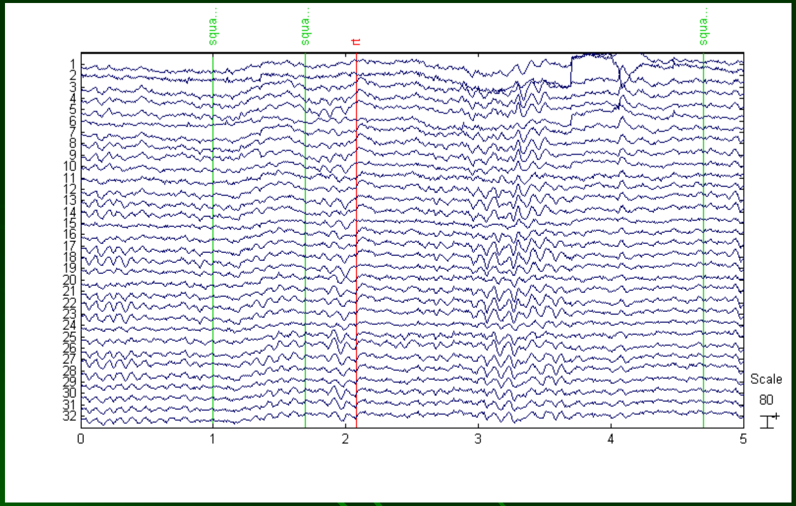


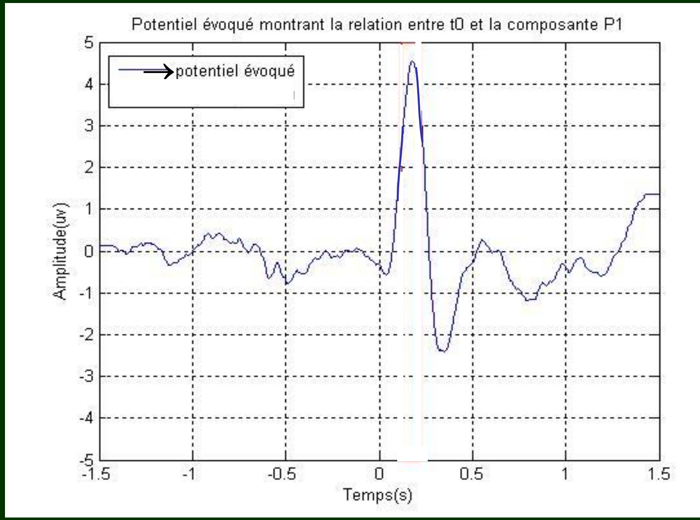


- **Position of the Electrodes**
- **32 Typical Recordings of the C1 Channel**

Computation of the Evoked Response Potentials





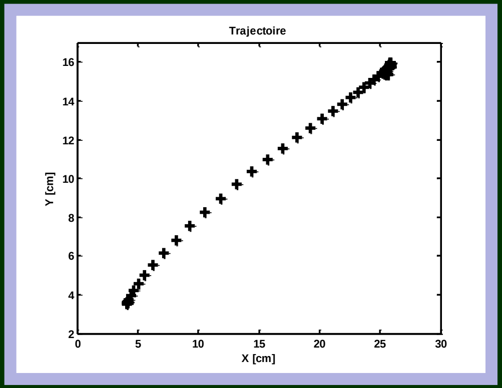


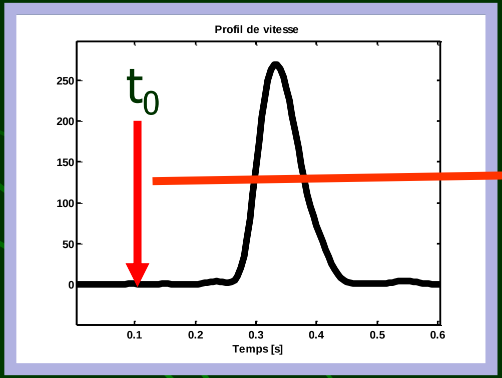
ERP can emerge through mean trace computation among trials for each electrode

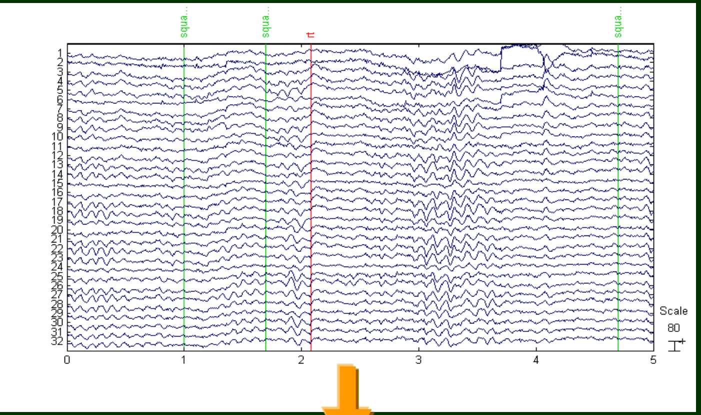
Typical Correlation

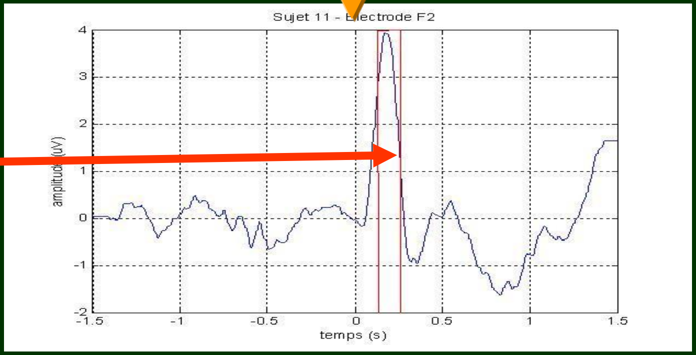
EEG signals

Kinematic data





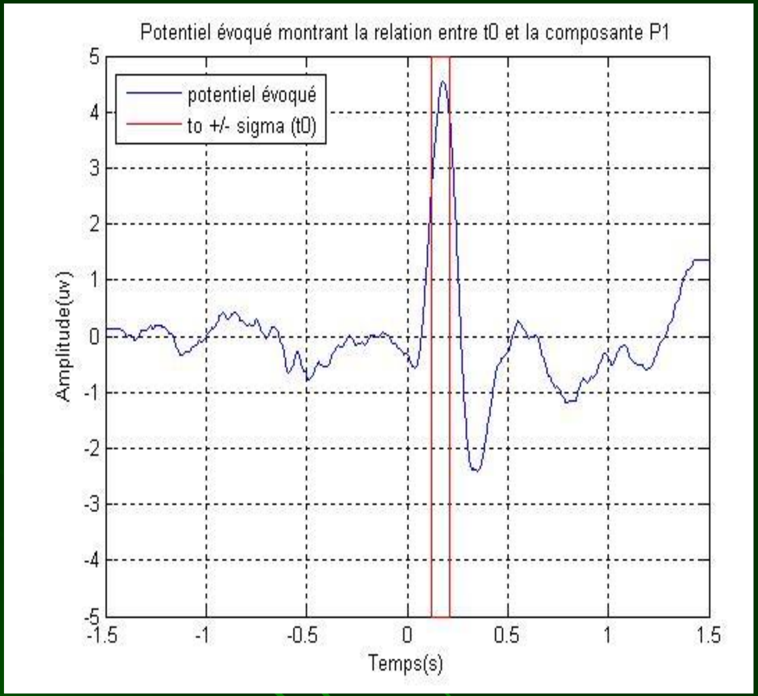




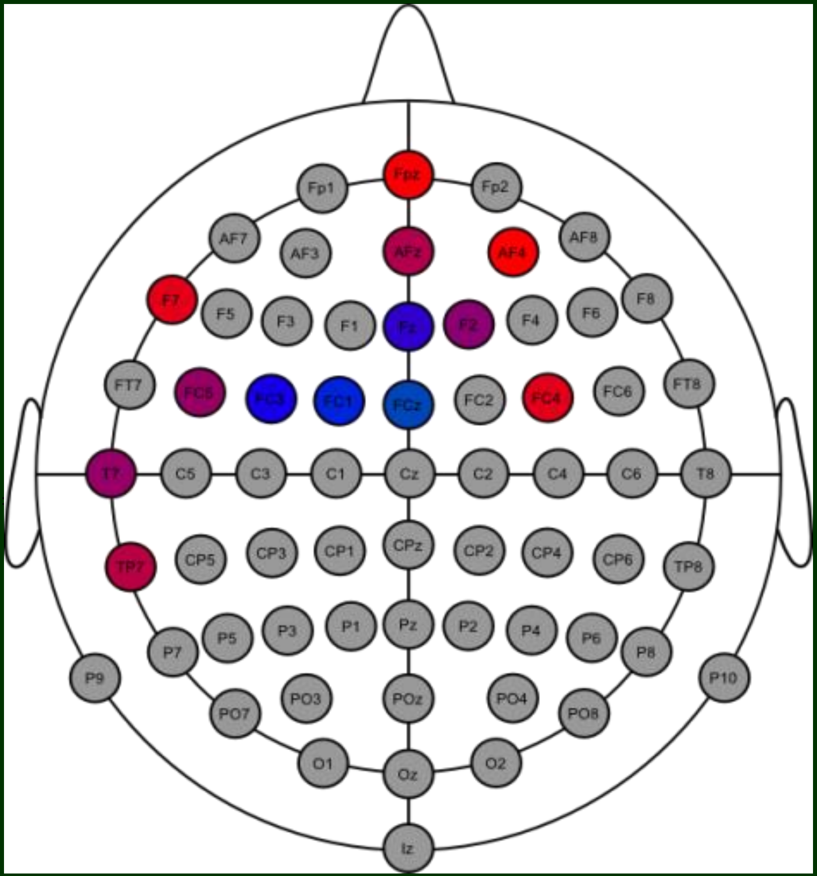
- **A visually evoked response potential emerges at t_0**

Summary of the Results

GOOD MATCH



t_{ERP}



Blue: $t = t_0$
Red $t > t_0$
Green: $t<t_0$

Theoretical
Problem No 2

Experimental Observation of the Proportional Effect Hypothesis

$$T_j = (1 + \varepsilon_j) T_{j-1}$$

PLAMONDON, R. DJIOUA, M., MATHIEU, P.-A., The Kinematic Theory of Rapid Human Movements: Experimental Observation of the Proportional Effect Hypothesis ,Human Movement Science, available online at http://dx.doi.org/10.1016/j.humov.2012.07.006, July 2012.

Apparatus

EMG Signal Acquisition System (GRASS)

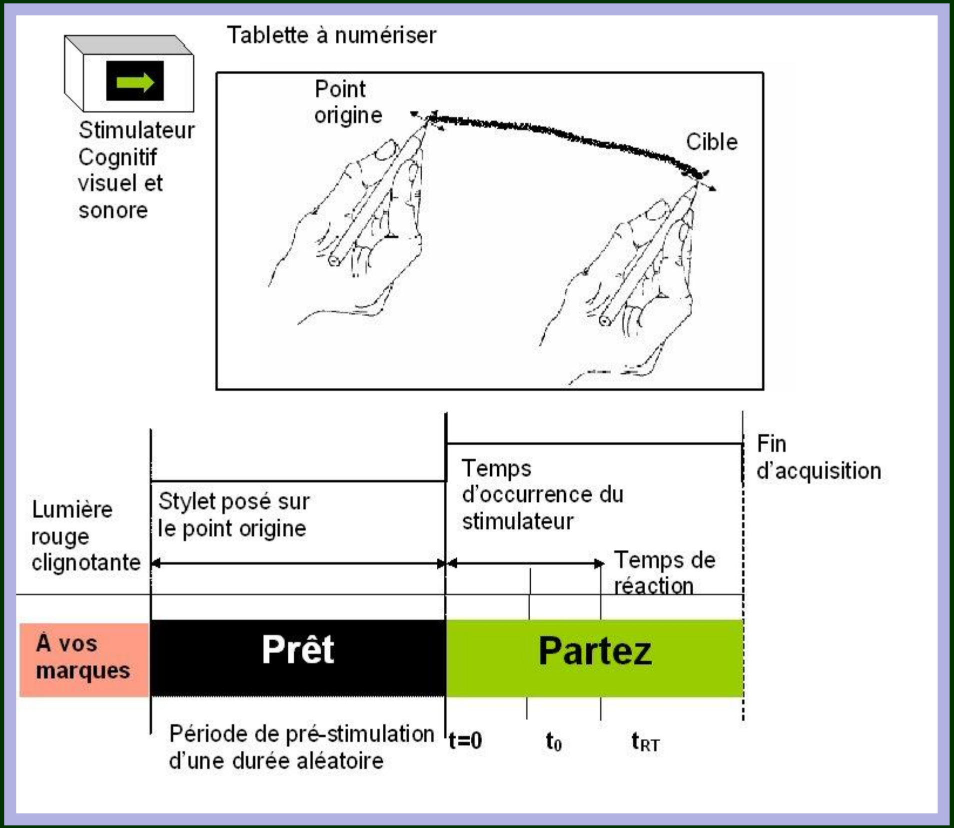
Amplifier

Data Acquisition System

SYNC signal

Experimental Protocol

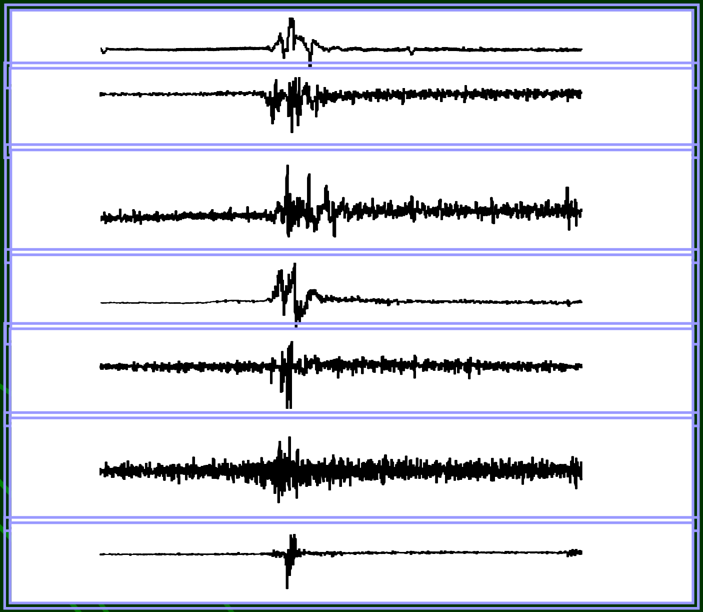
- 10 subjects
- right handed, good health,
- between 22 and 38
- Produce large strokes between a departure point and a target zone, following a reaction time protocol
- Auditory stimulus
- 20 valid trials

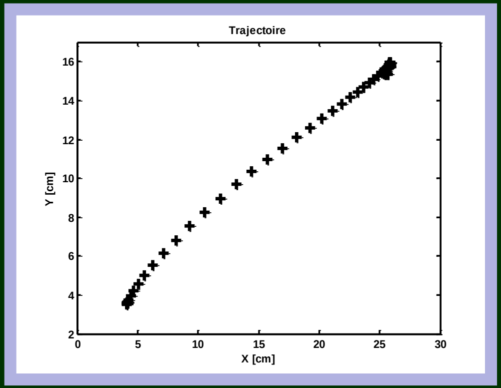


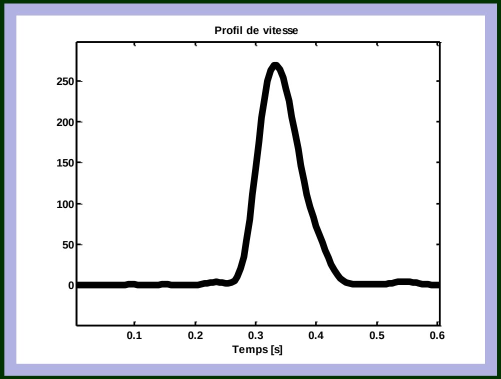
Typical raw data of biosignals

Kinematic data

Raw EMG signals

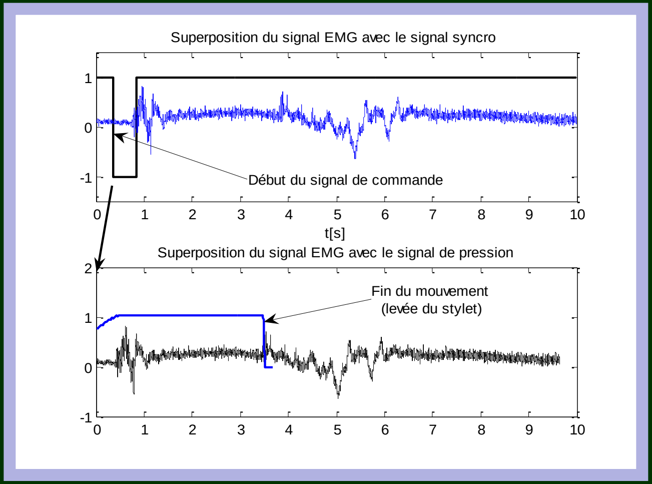






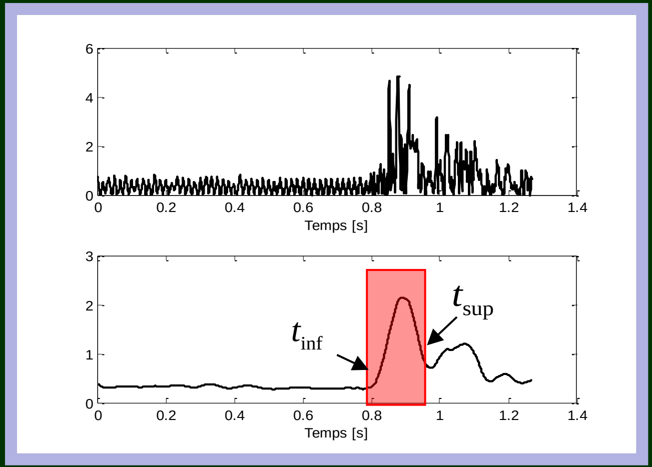
Preprocessing

1. Time Origin Definition



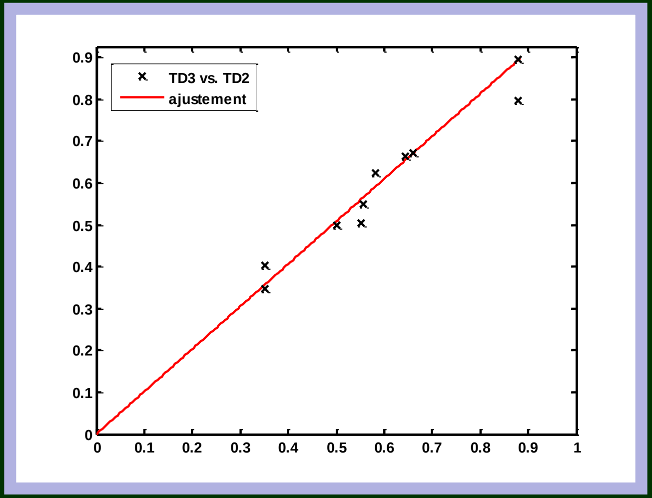
2. Computation of EMG envelopes and cumulative time delays

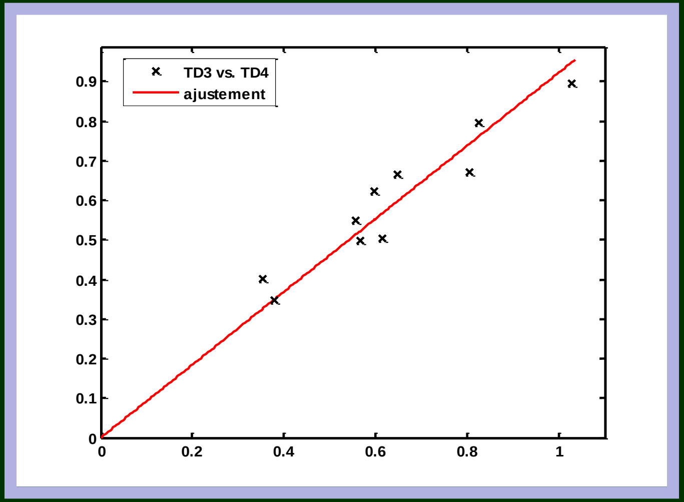
Savitzky-Golay Filter



$$T = \frac{\int_{t_{inf}}^{t_{sup}} t.Enveloppe_{EMG}(t)\,dt}{\int_{t_{inf}}^{t_{sup}} Enveloppe_{EMG}(t)\,dt}$$

Typical proportional regressions



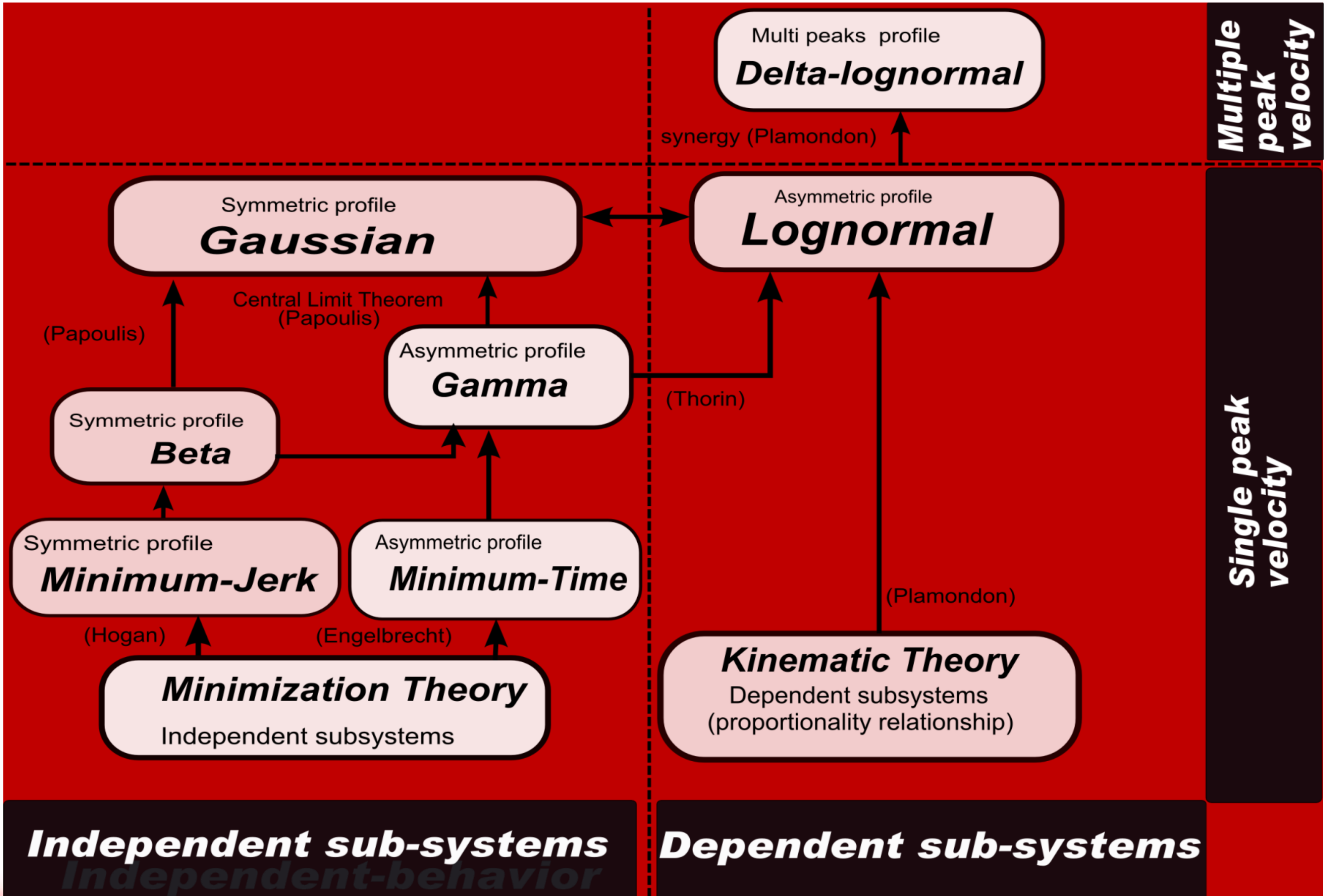


Correlation Coefficient r^2

X \ Y(X)	TD1	TD2	TD3	TD4	TD5	TD6
TD1	1.0	0.92	0.95	0.89	0.92	0.89
TD2	0.93	1.0	0.94	0.90	0.84	0.86
TD3	0.96	0.95	1.0	0.95	0.94	0.90
TD4	0.89	0.89	0.88	1.0	0.93	0.88
TD5	0.93	0.85	0.93	0.94	1.0	0.92
TD6	0.87	0.82	0.85	0.86	0.89	1.0

$r^2 > 0.82$

THEORETICAL COMPARISON



DJIOUA, M., PLAMONDON, R., "The Limit Profile of a Rapid Movement Velocity"
Human Movement Science, vol. 29, (2010), pp.48-61.

Challenge

Give me

a page

and a pen,

and I will move

time!

Défi,
translated by
Andrea Zanin.

Défi

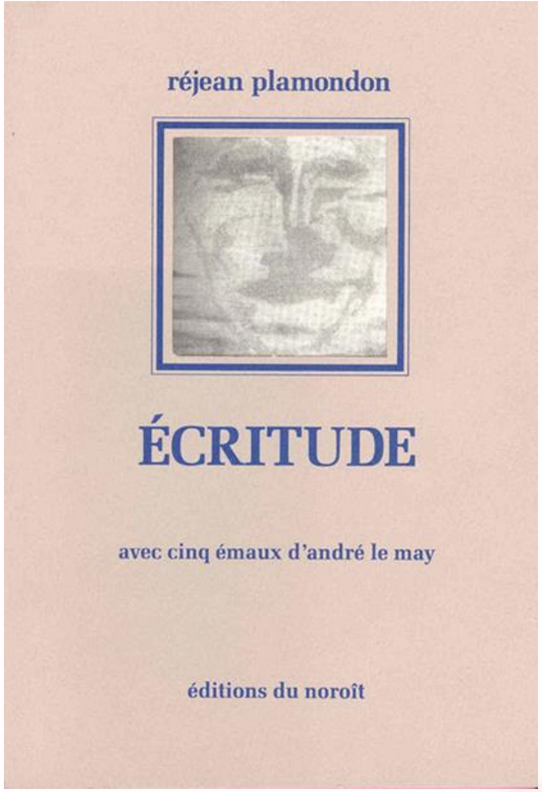
Donnez-moi

une feuille

et un crayon,

je soulèverai

le temps!



5 - Where Do We Go From Here ?

Back to poetry and principles

Counter-clockwise

Sometimes
I rebel
against minutes
to forget
all the seconds
waiting in line

The hours pass
without my noticing
that they were disguised
as days

Sometimes I spend months
taking a few moments
to give meaning
to my years

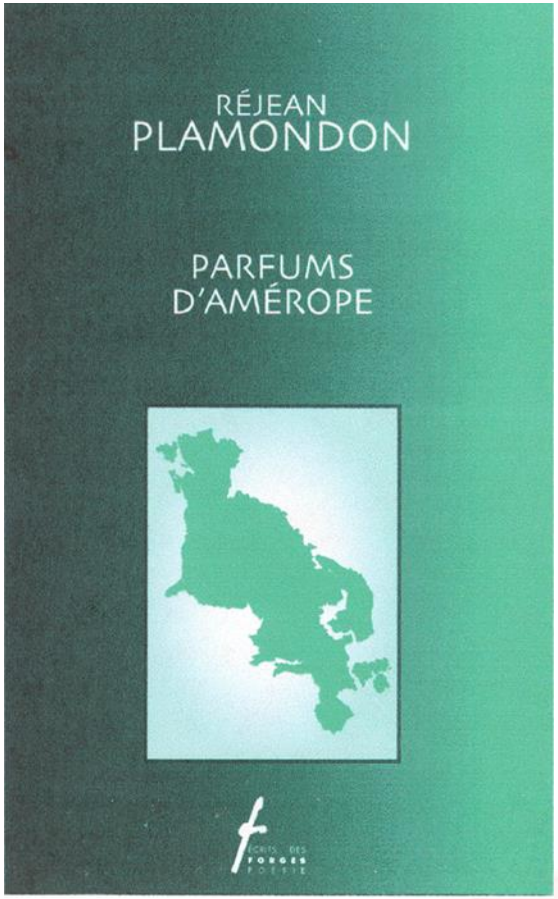
Anti-horaire,
translated by
Andrea Zanin.

Anti-Horaire

Parfois
je me révolte
contre les minutes
pour oublier
toutes ces secondes
en file d'attentes

Les heures passent
sans que je me rende compte
qu'elles s'étaient déguisées
en journées

Parfois les mets des mois
à prendre quelques instants
pour donner du sens
à mes années



Part 1:TOPICS

1. Lognormality Principle.
2. Learning: Moving toward Lognormality.
3. Aging: Moving away from Lognormality.
4. Sharing the Tools-Expanding Knowledge.
5. The Tip of an Iceberg.

1 – Lognormality Principle

Lognormality Principle

The lognormal velocity patterns, which are the results of an asymptotic convergence, can be interpreted as reflecting the behavior of subjects who are in perfect control of their movements.

Corollary

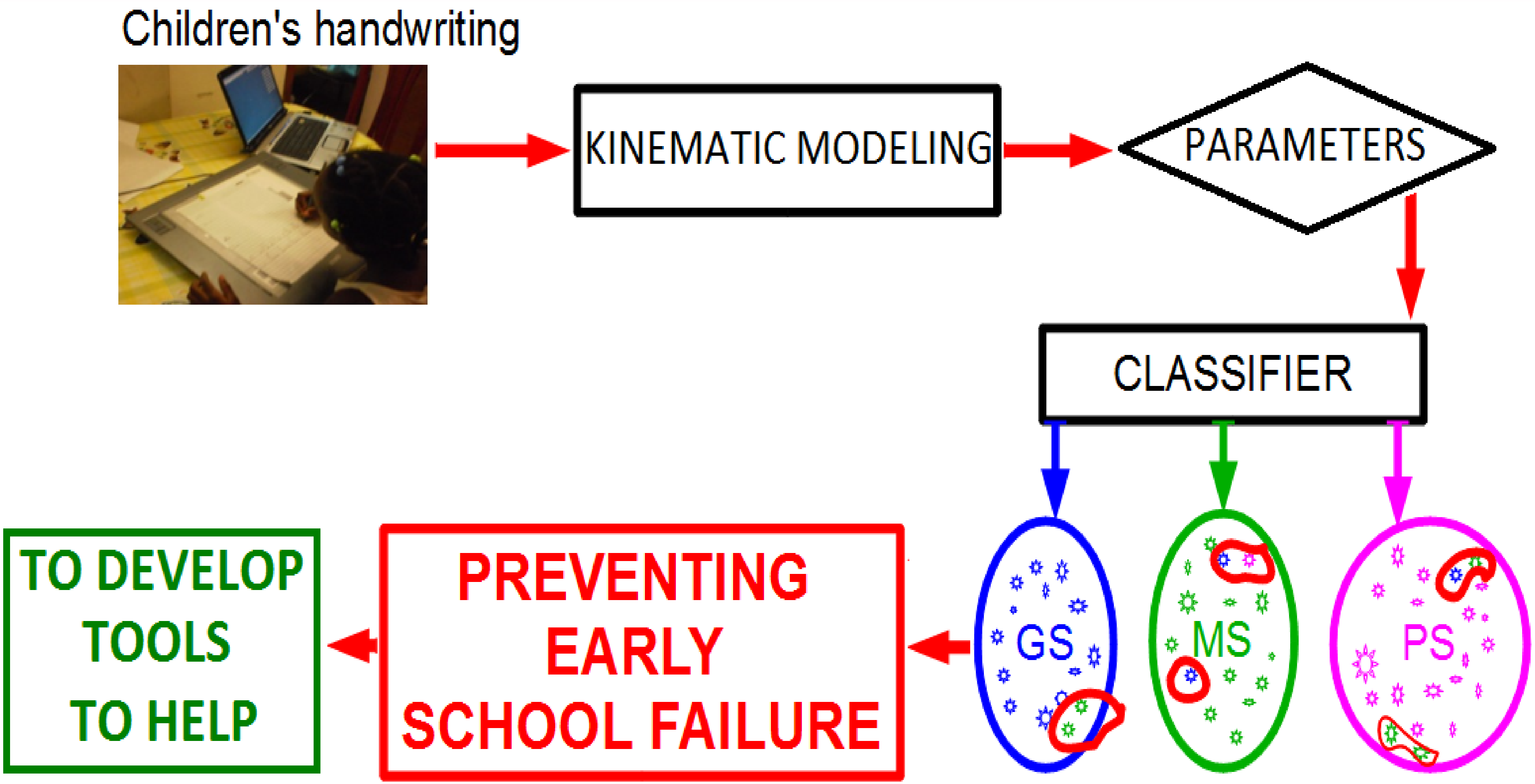
If we specifically focus on the basic mathematical convergence toward lognormality, handwriting learning, on the one hand, can be interpreted as a migration toward an ideal representation of perfectly mastered movements. On the other hand, aging and health problems should reveal a progressive departure from lognormality. In between, writers are taking advantage of lognormality.

2 - Learning:

Moving toward Lognormality

- Cooperation with Céline Rémi and Thérésa Duval, Université des Antilles et de la Guyanne.
- DUVAL, T., RÉMI, C., PLAMONDON, R., O'REILLY, C.,, On the Use of the Sigma-Lognormal Model to Study Children Handwriting, , Proc.16th Biennial Conf. of the Graphonomics Society, Nara, Japan, June 10-14 2013, pp26-29.

GOAL



Children's handwriting
KINEMATIC MODELING
PARAMETERS
CLASSIFIER
GS
MS
PS
PREVENTING EARLY SCHOOL FAILURE
TO DEVELOP TOOLS TO HELP

FUNDAMENTAL QUESTIONS

Context $\longrightarrow$ preliminary study to test the relevance of the Sigma-lognormal model on young writers

1. ***Is the Sigma lognormal model efficient to synthesize children's handwriting with a good quality?***

2. ***Is the Signal to Noise Ratio (SNR) efficient to classify young writers?***

3. ***Is the number of lognormals (nblog) efficient to classify young writers?***

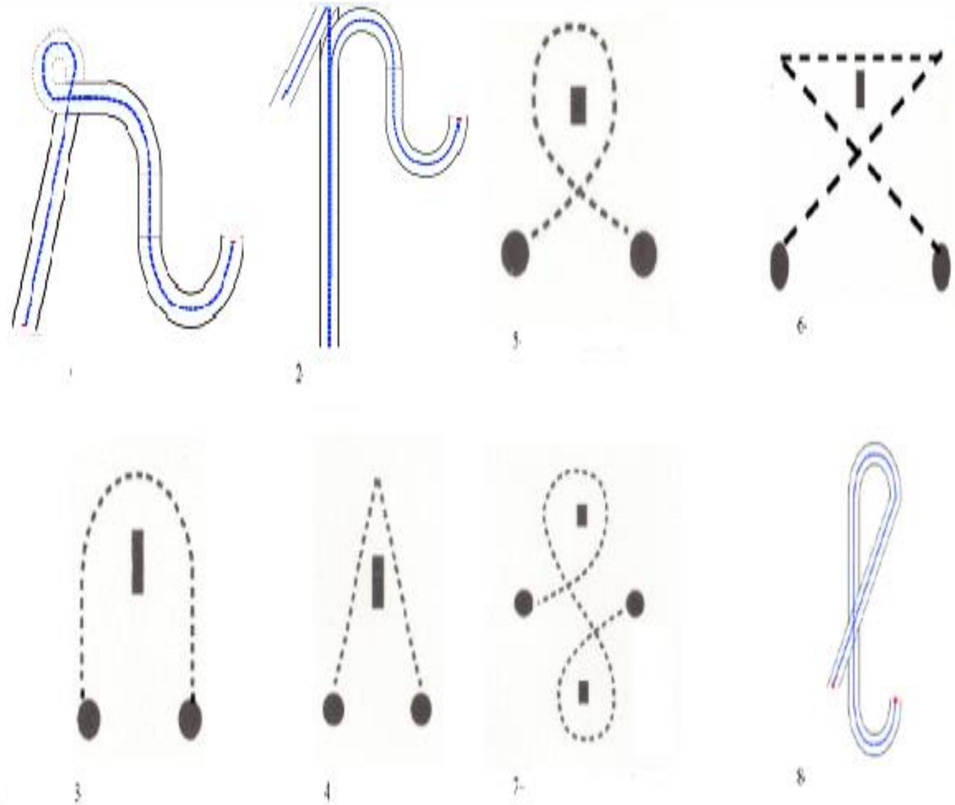
DATABASE

Number of days : 5

Section	Large (GS)	Medium (MS)	Small (PS)	Total
Children	27	17	22	66
Age (years)	5	4	3	

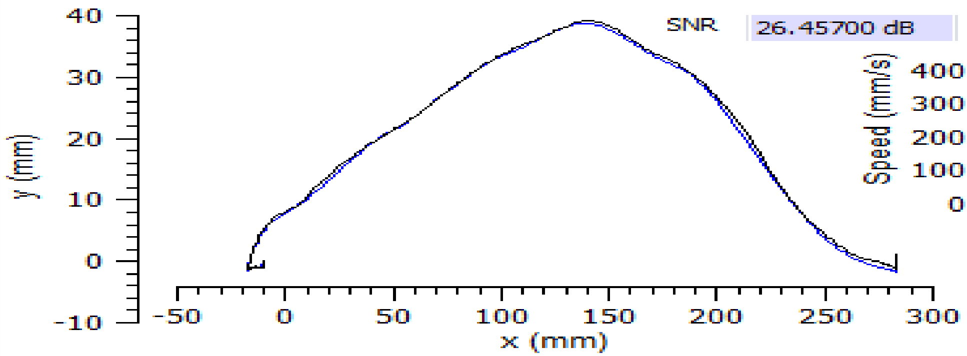
44 trajectories by children $\longrightarrow$ 2640

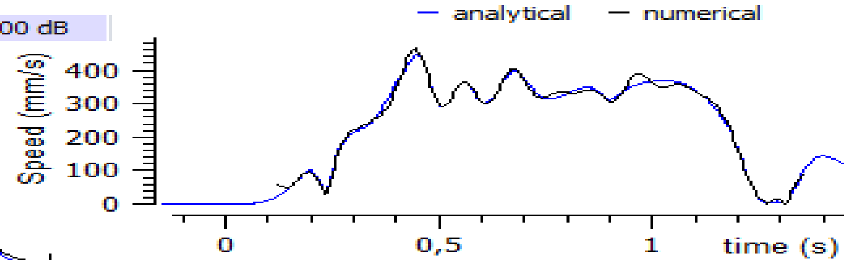
TOOLS,PROTOCOL OF ACQUISITION



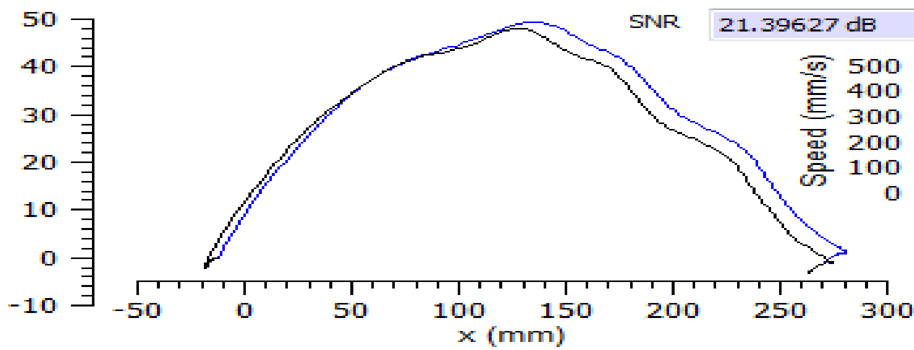
Number of patterns : 11

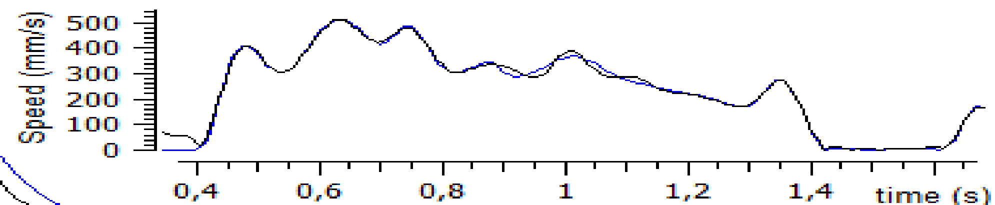
Fitness of the reconstruction: SNR



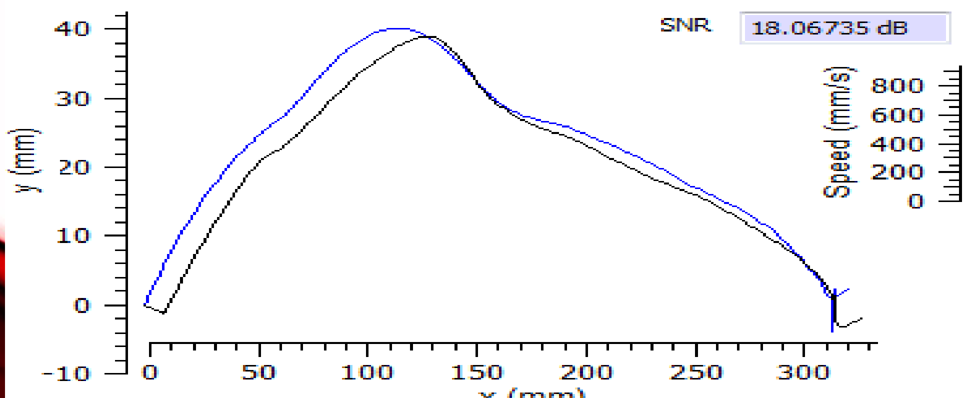


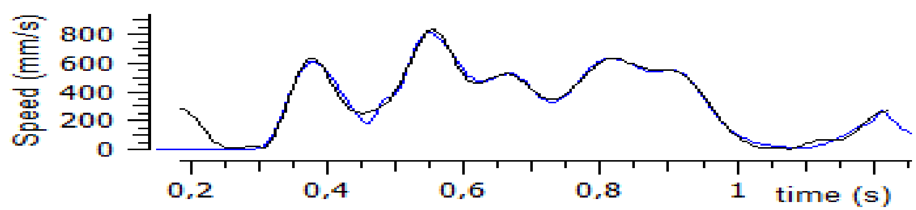
SNR ≥ 25
***Excellent Quality**
***Master level**





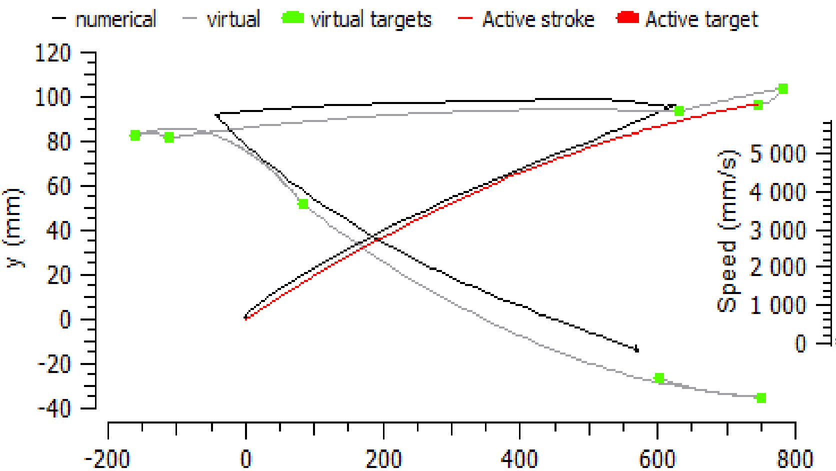
20 ≤ SNR < 25
***Correct quality**
***Intermediate level**

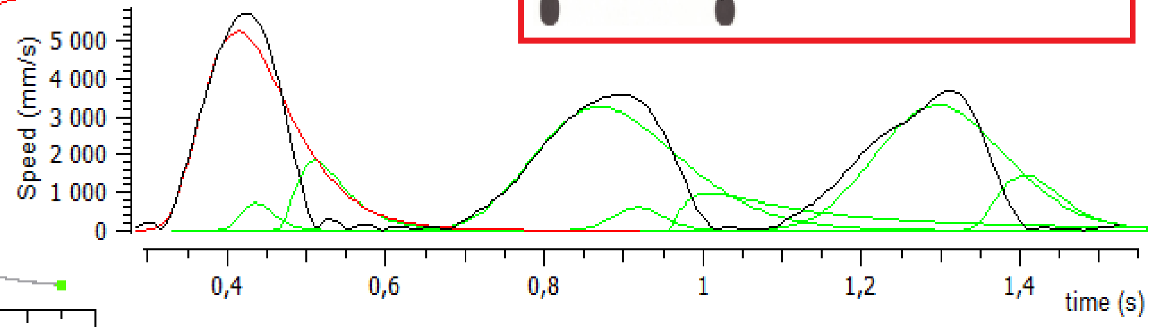




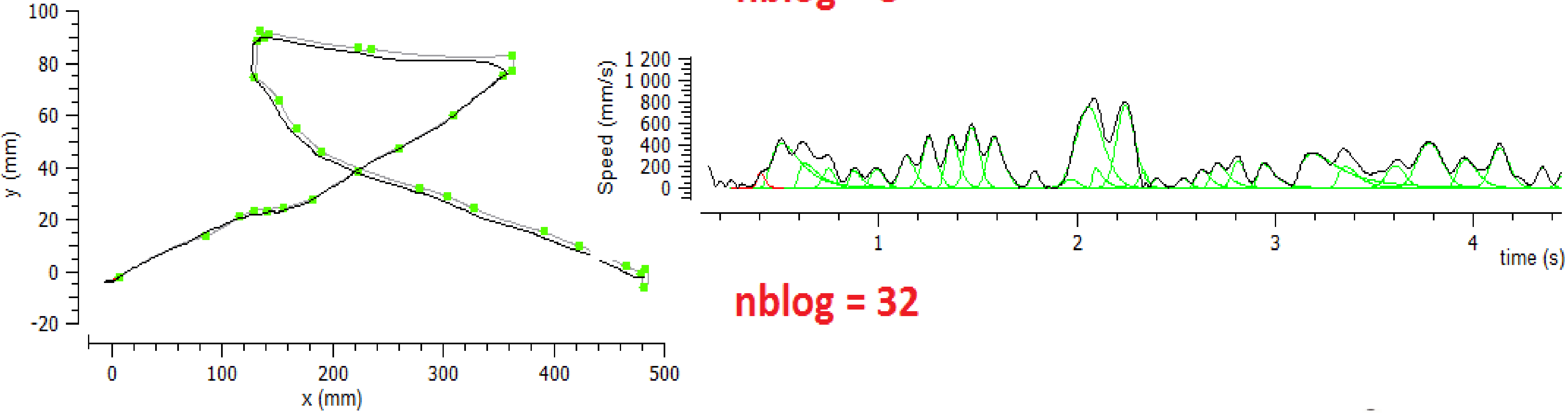
SNR < 20
***Weak Quality**
***Beginner level**

Fluidity of the movements: Nblog





nblog = 8



nblog = 32

Summary

- The three indicators, SNR, Nblog and SNR/Nblog, as extracted from the sigma-lognormal model, can effectively point out the converging behavior toward lognormality of young writers producing simple traces.
- When considering the levels of learning and scriptural mastery (Master, Intermediate and Beginner), it is clear that at least three classes can be defined (AD, GS and MS-PS) for which the older is the writer, the better is its level of mastering a lognormal writing behavior.

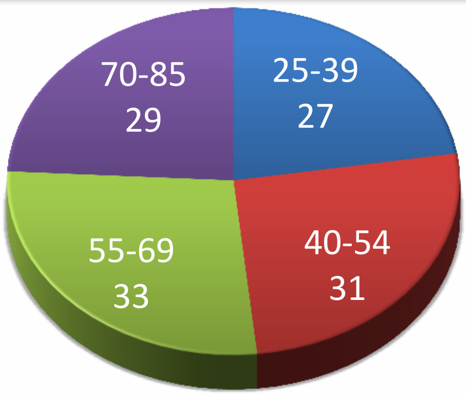
3 – Aging:
Moving away from Lognormality

Back to a Previous Study on Brain Stroke Risk Factors Assessment

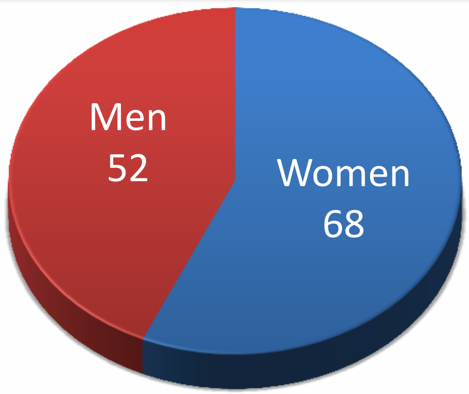
5 Data

Sample (120 subjects)

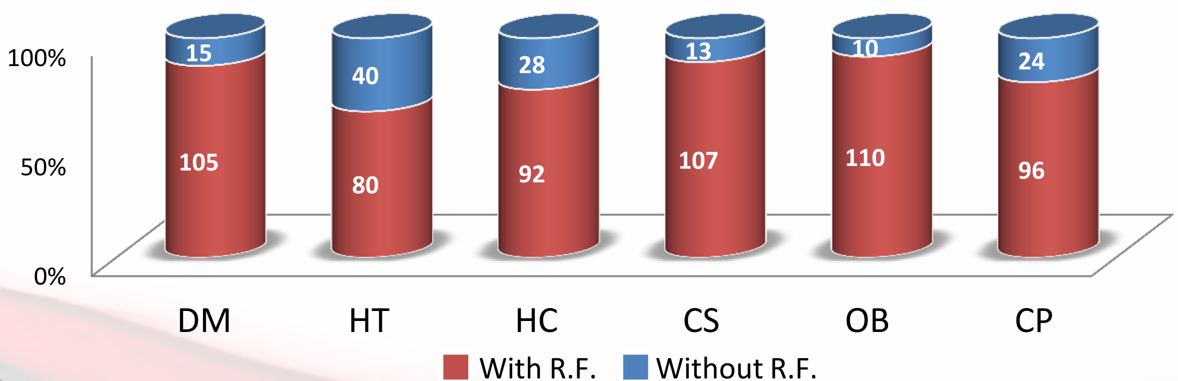
Age



Gender



Brain stroke risk factors (R.F.)



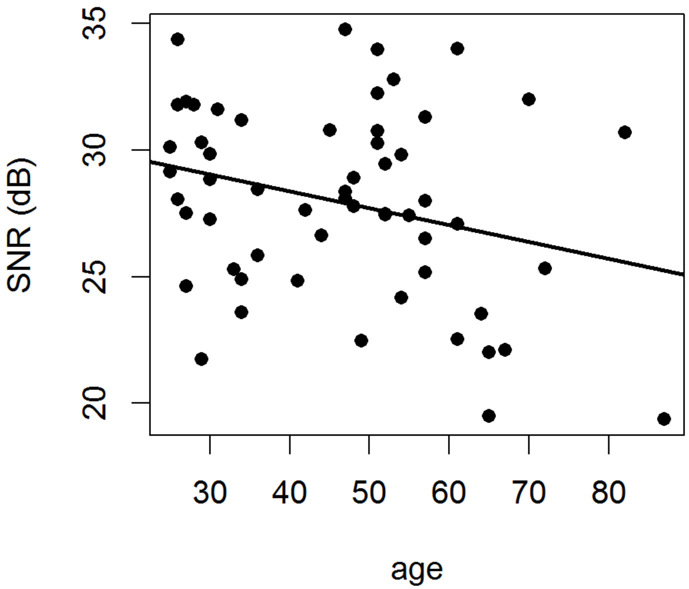
DM: Diabetes mellitus
HT : Hypertension
HC : Hypercholesterolemia
CS : Cigarette smoking
OB : Obesity
CP : Cardiac problems

PREVIOUS STUDY SUMMARY

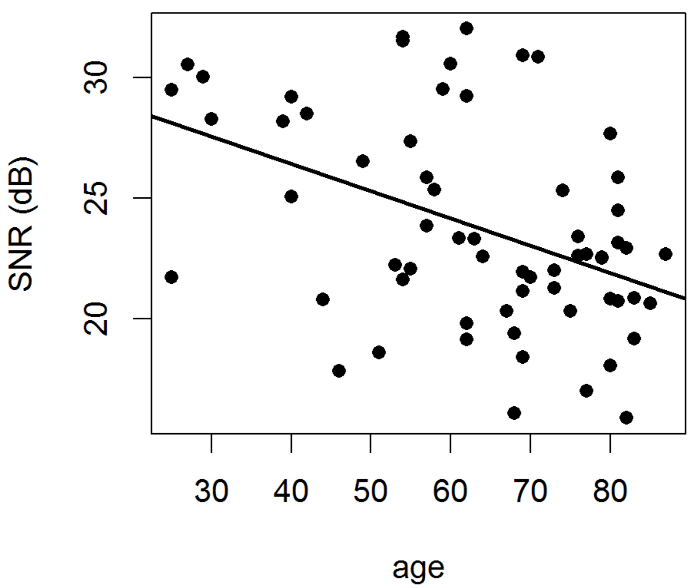
- As area under the ROC curve of 0.68 to 0.82 have been obtained on the prediction of six of the principal stroke risk factors using only information extracted from the movements, there is a definitive relationship between the presence of stroke risk factor and the characteristics of human movements. Although, a large part of it may be attributed to the effect of the age and the gender, there are convincing evidences that these two factors does not account for all of it. Therefore, human movements seem to contain supplementary information related to the susceptibility of eventually suffering from a stroke which is neither attributable to the age nor the gender.

PLAMONDON, R.' O'REILLY, C,. OUELLET-PLAMONDON, R., Strokes against Strokes-Strokes for Strides, In Press, Pattern Recognition, 2013, availlable online at http://authors.elsevier.com/sd/article/S0031320313002082

Reaching Movements



No risk factor



With risk factors

Triangle Drawing

Sample	Response variable	Intercept (p-value)	Age (p-value)	Gender (p-value)	Interaction (p-value)
NFR	SNR	**21.33 (<2e-16)**	0.01826 (0.22)	0.1664 (0.88)	-0.01232 (0.59)
	nbLog	**4.982 (1.76e-05)**	**0.05695 (0.0079)**	2.192 (0.17)	-0.04978 (0.13)
	SNR/nbLog	**3.792 (3.91e-14)**	**-0.01514 (0.042)**	-0.6357 (0.25)	0.01215 (0.29)
WFR	SNR	**21.67 (<2e-16)**	0.01264 (0.20)	0.4452 (0.71)	-0.005349 (0.77)
	nbLog	**6.680 (1.25e-04)**	**0.06023 (0.017)**	-0.2590 (0.93)	-0.003594 (0.94)
	SNR/nbLog	**3.148 (8.26e-14)**	**-0.01333 (0.0085)**	0.09853 (0.87)	6.530e-05 (0.99)

Summary

The hypothesis that age and health problems result in a divergence from lognormality is supported by these two studies.

For the delta-lognormal reconstruction of rapid single strokes, the SNR decreases with age as a result of motor control degradation is generally supported.

For the sigma-lognormal reconstruction of the triangles, taking into account the number of lognormal components used to model the movement makes apparent the divergence effect.

For Further Information

Plamondon, R., O'Reilly, C., Rémi, C., Duval, T.,
"The lognormal hand writer:
learning, performing and declining."
Frontiers in Cognitive Science, (2013)
Special Issue on Writing Words: from Brain to Hand(s),
S. Kandel and M. Longcamp, Eds.
A Nature Publishing Group Journal.

4 - Sharing the tools
Expanding knowledge

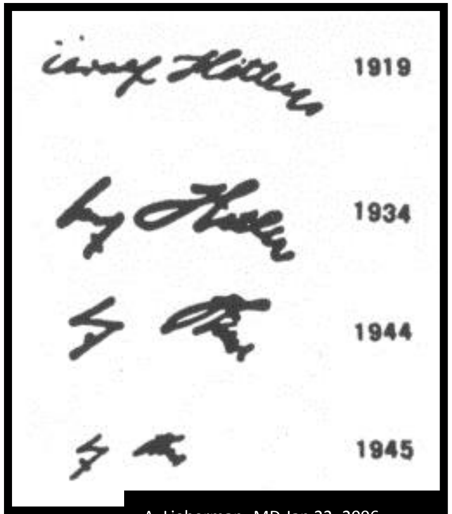
Overview of Some Collaborative Quests

Parkinson Disease

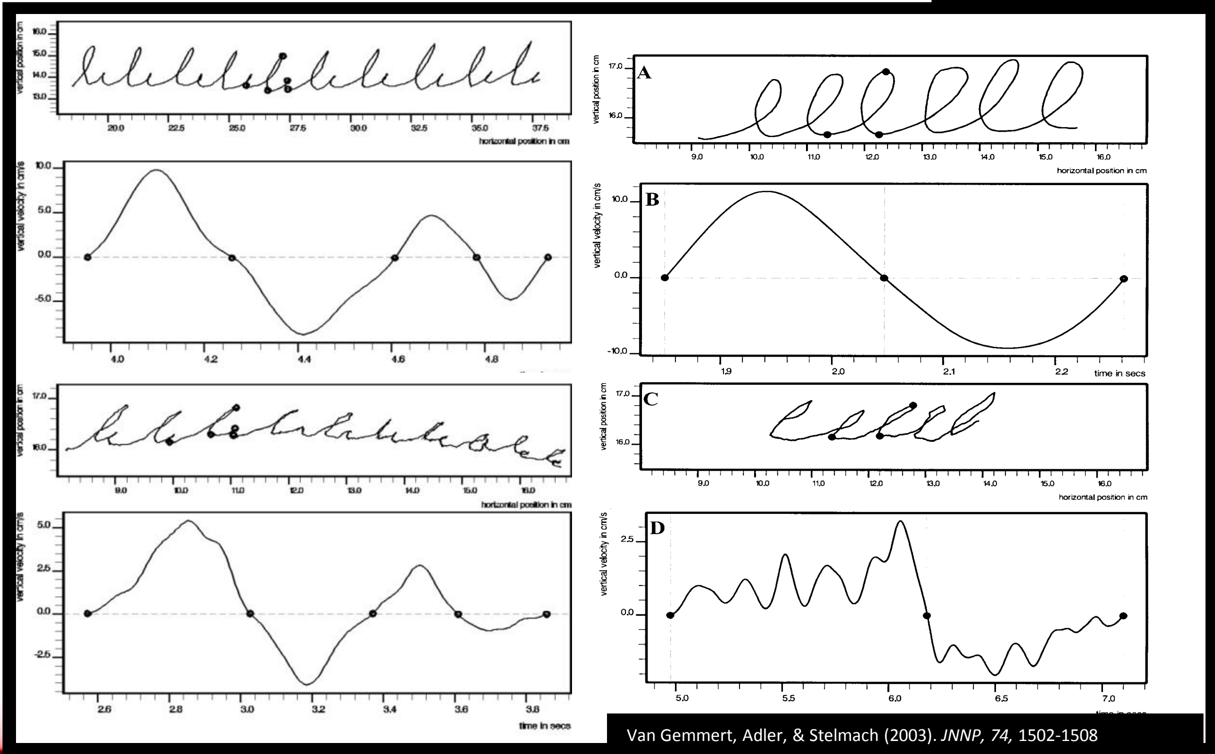
- A cooperation with Aren van Gemmert, Zhujun Pan and Christopher Aiken, Department of Kinesiology, Louisiana State University.
- VAN GEMMERT, A, PLAMONDON, R. O'REILLY,C, Using the Sigma-lognormal model to investigate handwriting of individuals with Parkinson's disease, Proc.16th Biennial Conf. of the Graphonomics Society,Nara, Japan, June 10-14, 2013,pp. 119-122.

Micrographia

- PD patients are less able to increase stroke size without marked changes in stroke duration due to reduced force modulation efficiency.
- As result of this reduced ability, the isochronic size range is reduced in PD patients
- PD patients have difficulty to write larger than 1.5 cm stroke sizes

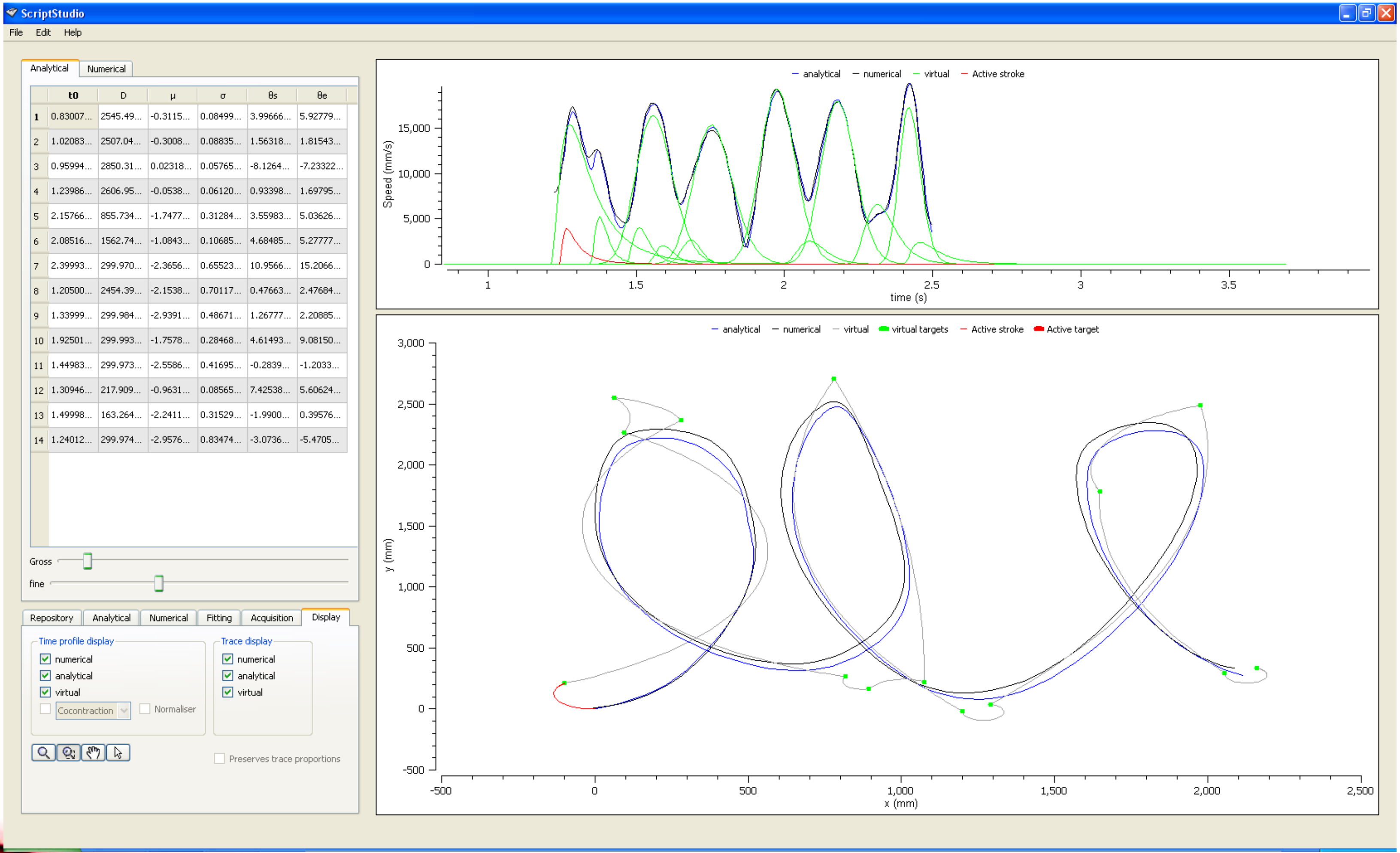


A. Lieberman, MD Jan 22, 2006

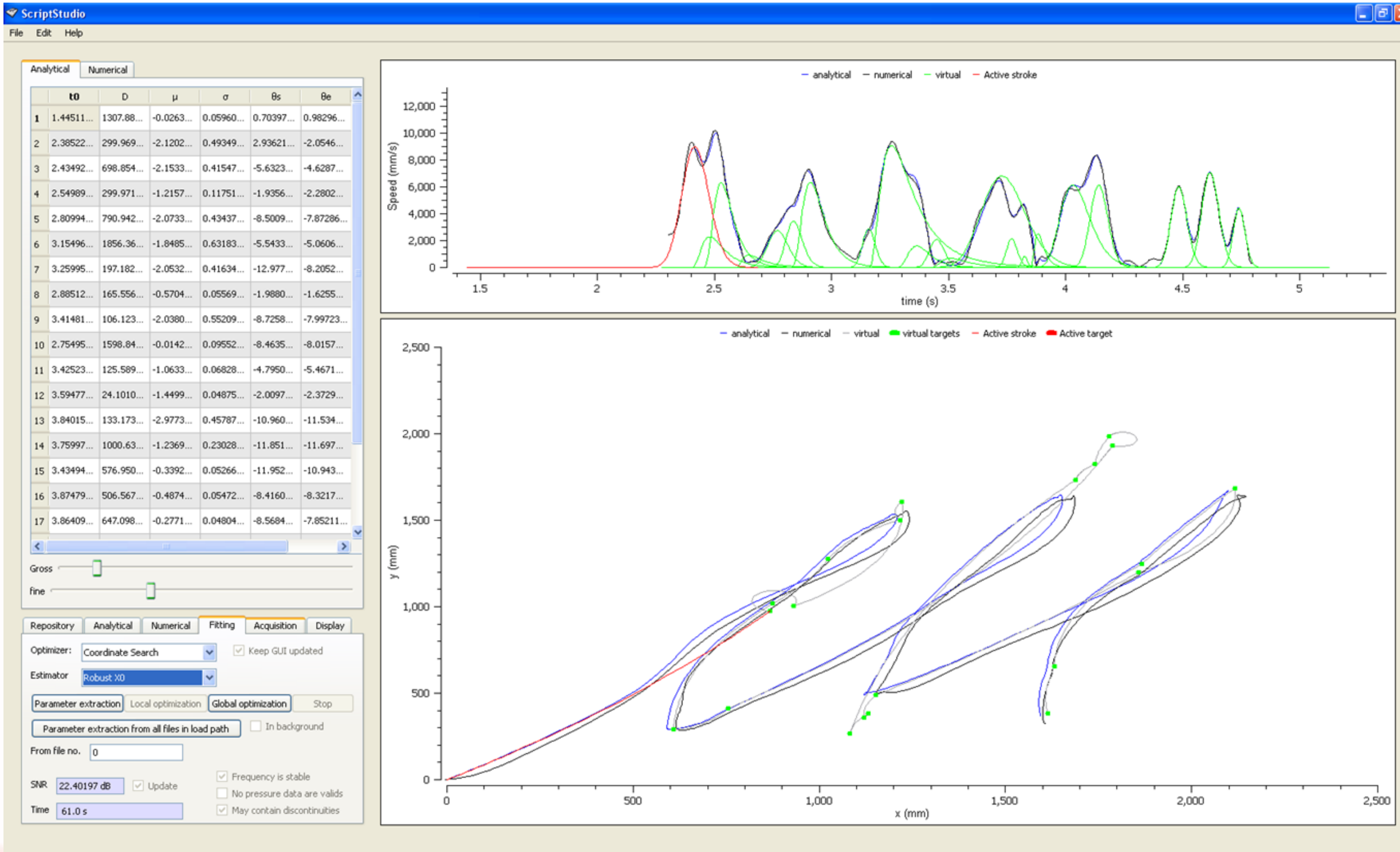


Van Gemmert, Adler, & Stelmach (2003). *JNNP, 74,* 1502-1508

Results (Older control)



Results (Parkinson)



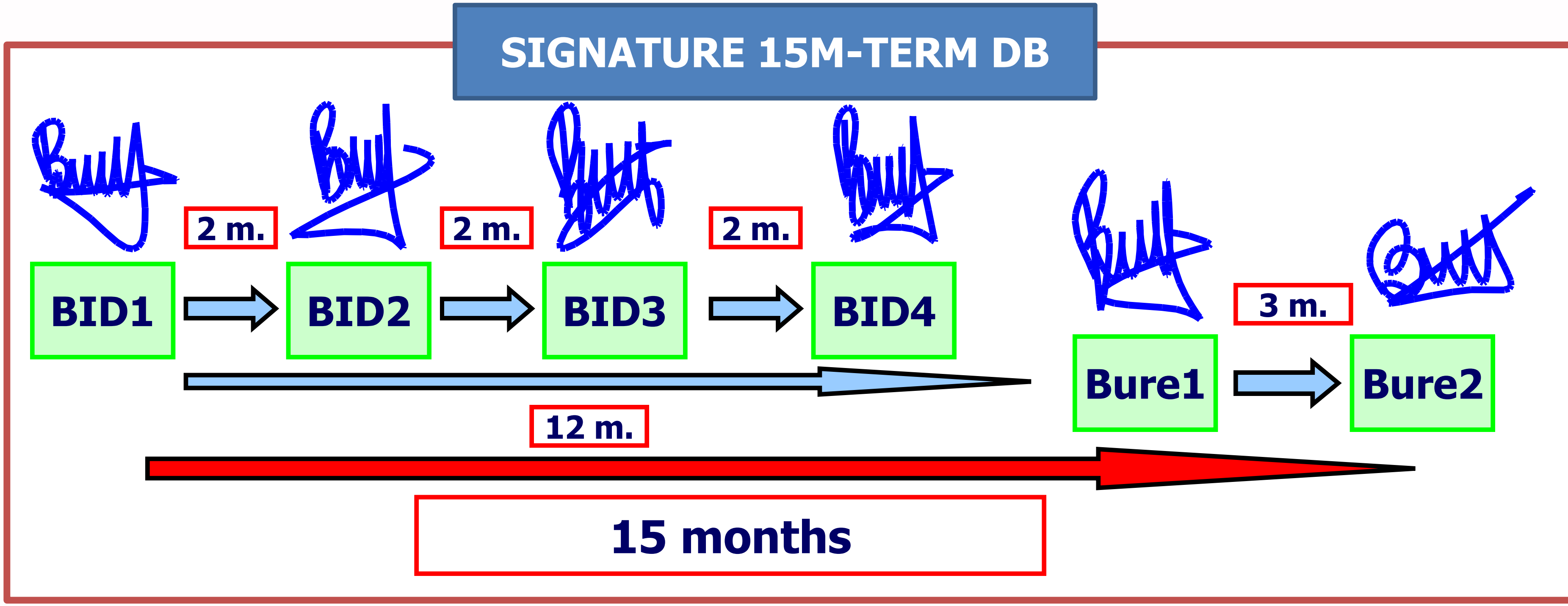
Preliminary Results

- PD patients use significant more Lognormals than controls
  - PD patients have difficulty to precisely control agonist and antagonist resulting in more co-contraction => smaller movements.
- PD patients have trouble to efficiently recruit the order of lognormals to produce smooth movements
- Sigma-Lognormal model useful to find primitives in motor movements of Parkinson Disease patients.

Signature vs Time

- A Cooperation with Javier Ortega-Garcia, Julian Fierrez, Javier Galbally and Marta Gomez-Romero, Biometric Recognition Group, ATVS,Universitad Autonoma de Madrid.
- GOMEZ-BARRERO, M., Galbally, J., Fierrez, J., Ortega-Garcia, M., Variations of Handwritten Signatures with Time: a Sigma-lognormal Analysis, Proceedings of the 6th IAPR International Conference on Biometrics, Madrid, Spain, June 4-7, 2013, PS3.1-PS3.6.

Variations of Handwritten Signatures with Time: A Sigma-Lognormal Analysis

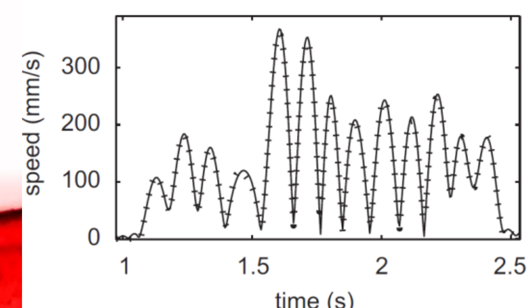


Sigma Log-Normal Representation
$(N, t_0, D, \theta_d, \theta_f, \mu, \sigma)$

Exp 1: Variation of the same signer with time

Exp 2: Variations between signers of different age groups

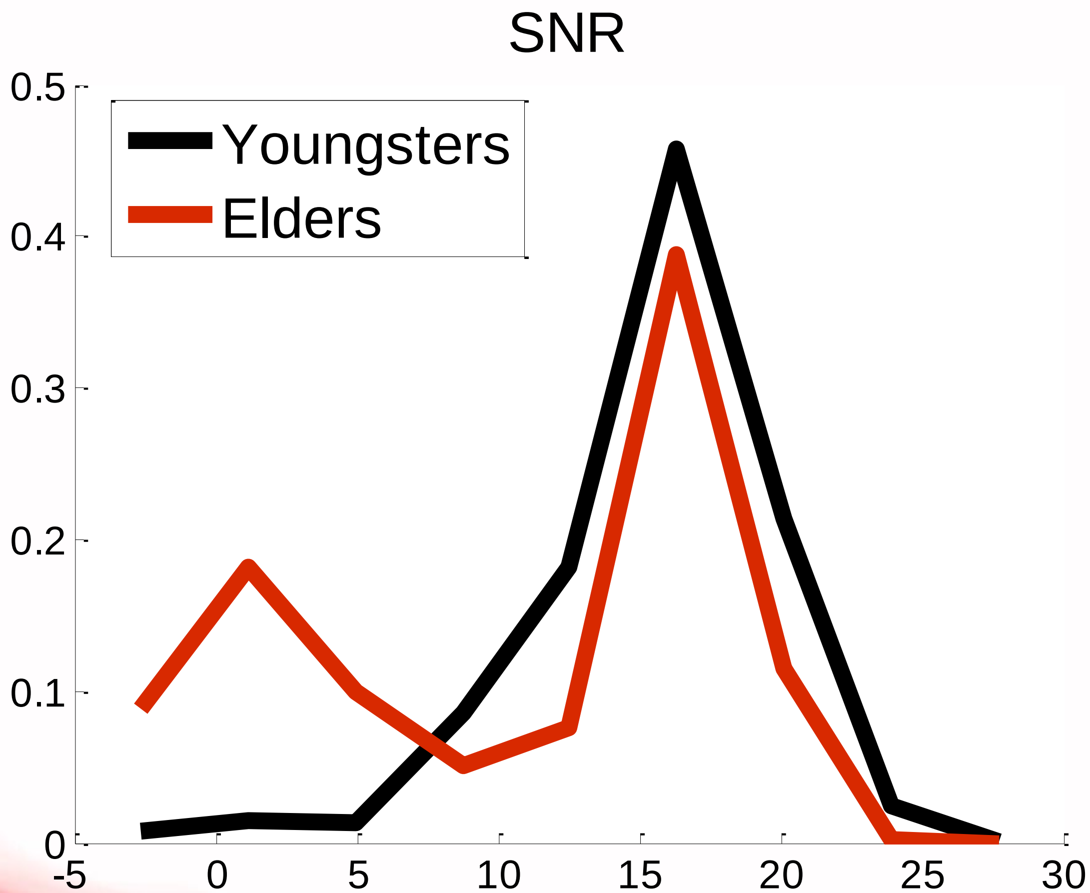
Statistical Measures



Same Signer:Variation with Time

- **nbLog** shows an growing trend: as we get older, the number of strokes increases.
- **Δt_0** shows a decreasing trend: similarly, those movements tend to be shorter.
- This is consistent with the lognormality principle.

Different Signers: Variation with age



Age Groups

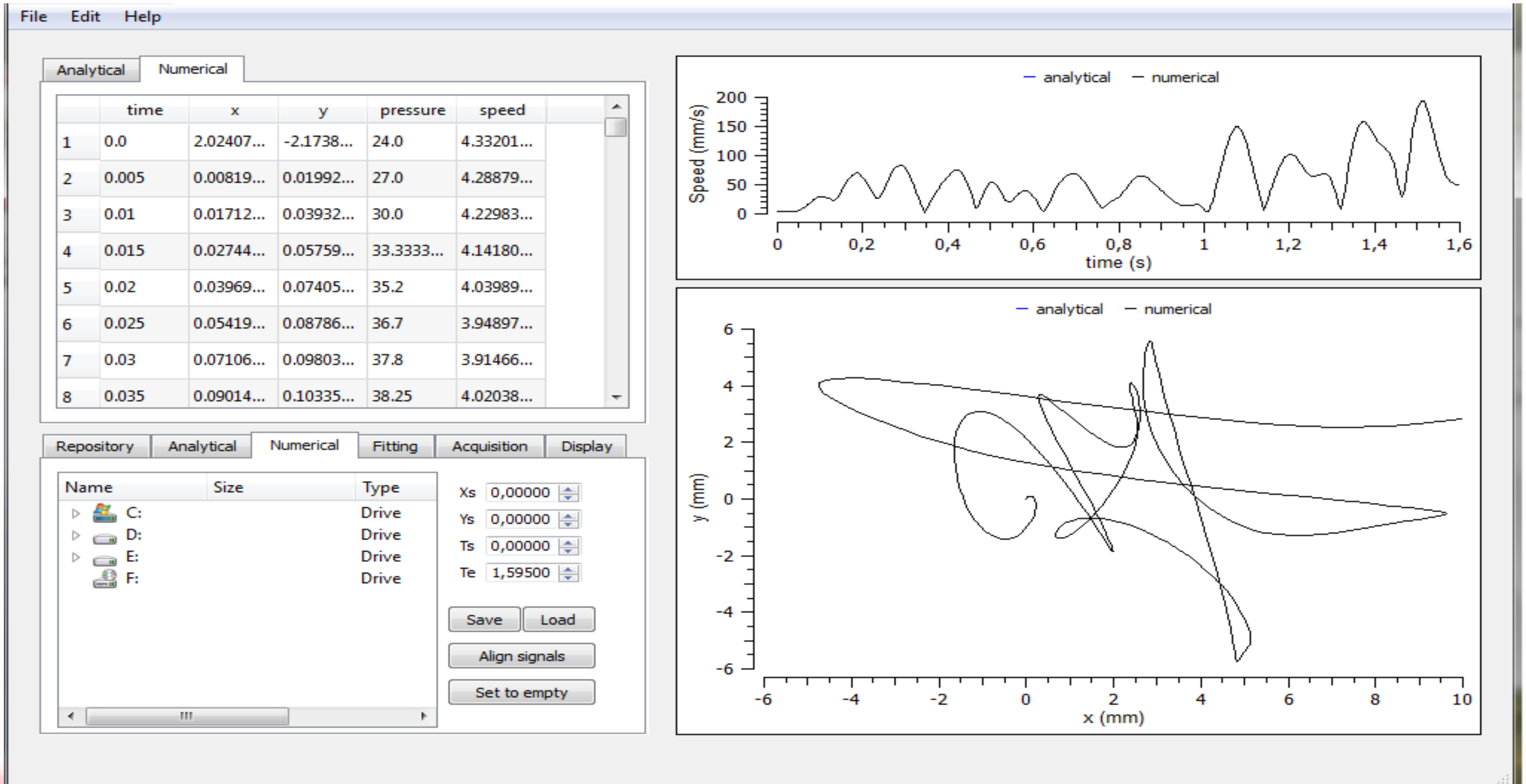
- Longer strokes (larger **D**) are more common in youngsters.
- **N** and **Δt_0** reinforce the results obtained in the previous experiments.
- More and smaller movements imply a lower logtime delay (**μ**) and a similar logresponse time (**σ**).
- The **SNR** distribution has a significantly bigger peak on lower values for the elder group: age makes us move away from lognormality (smaller **SNR**).
- The ratio **SNR/N** accentuates the trend shown by **SNR** and **N**: age results in a higher level of trembling (more small lognormals), and a bigger deviation from lognormality.

Signature Stability

- A cooperation with Guiseppe Pirlo, Donato Impedovo, Annamaria Cozzolongo, Roberta Gravinese, Andrea Rollo, Department of Computer Science, University di Bari, Italy.
- G. Pirlo, D. Impedovo, A. Cozzolongo, R. Gravinese, A. Rollo, R. Plamondon, C. O'Reilly, Stability of Dynamic Signatures: From the representarion to the generation domain, Proceedings of the International Workshop on Emerging Aspects in Handwritten Signature Processing , Naples, Italy, In Press, September 2013.

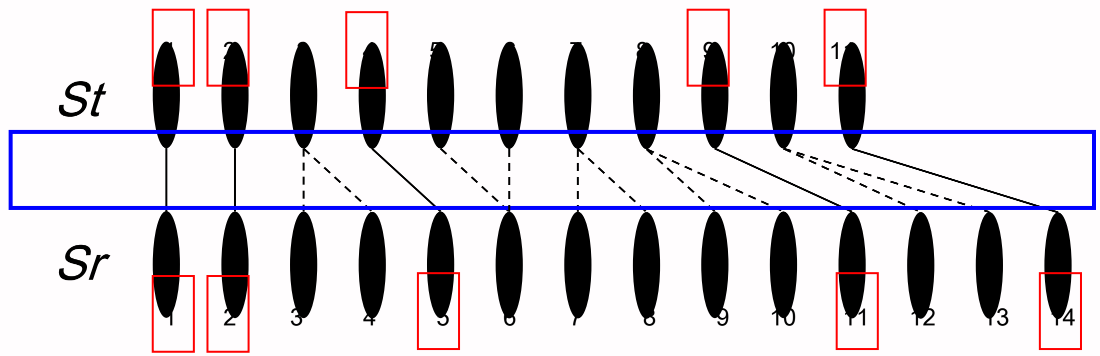
FEATURE EXTRACTION

ScriptStudio: software for signature acquisition and parameter extraction



Stability Analysis by DTW

A Direct Matching Point (DMP) is a point of St which has a one-to-one coupling with a point of Sr. A DMP identifies a part of the signal in which no strong difference exists between the two genuine specimens.



Example of DMP (red-squared points) of St and Sr (note that, in this case, St is described by 11 lognormal functions and Sr by 14 lognormal functions)

Stability Analysis by DTW

Let be:

- St a genuine signature
- Sr , r=1,2,…, R a set of R genuine signatures,

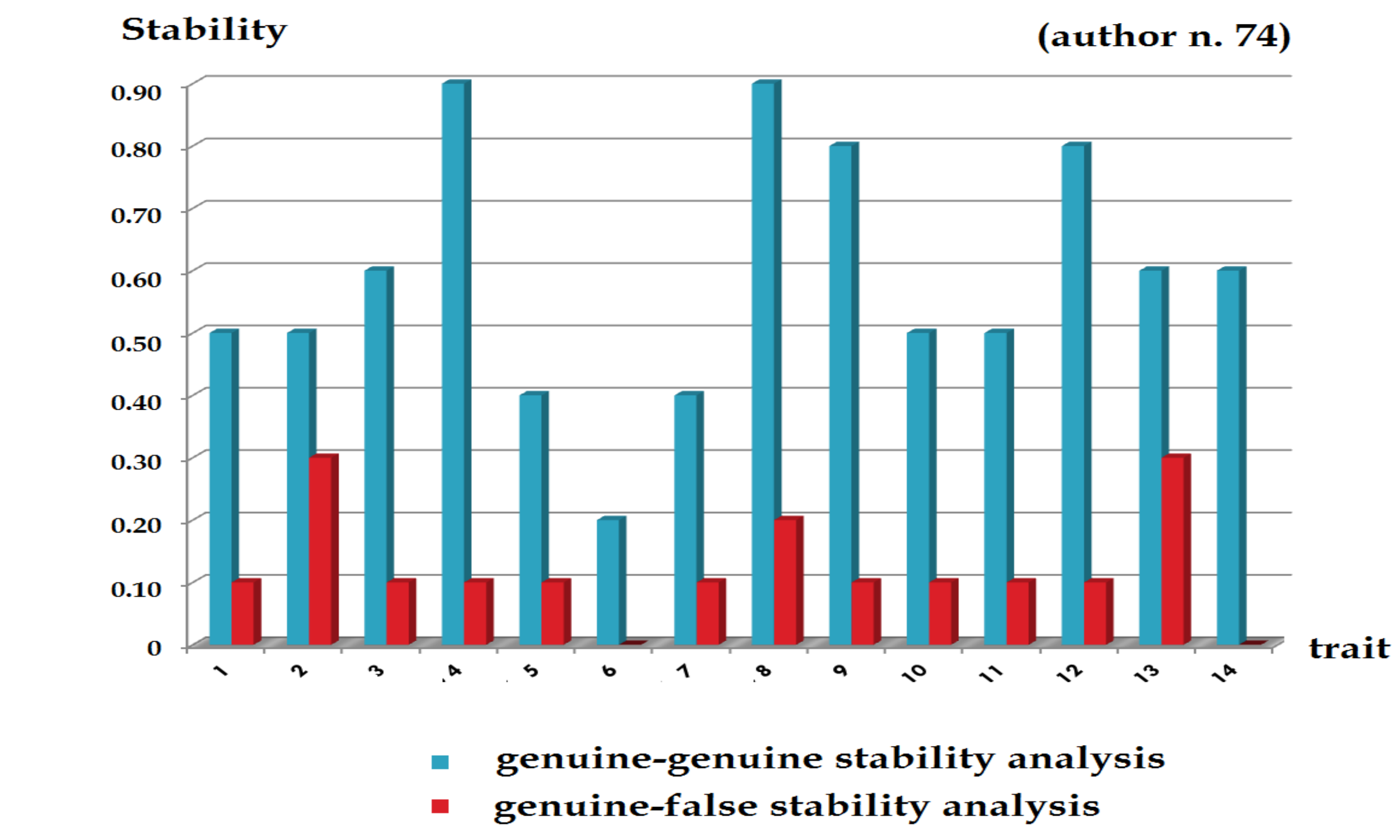
We first compare St against each Sr , r=1,2,…R.

The Local Stability of the i-th trait of St is defined as

Local Stability St (i) = (1/R) * Number of times trait i is DMP

Of course Local Stability ranges from o (low stability) to 1 (high stability).

EXPERIMENTAL RESULTS (trait-oriented stability analysis)



Alzheimer Handwriting Analysis

- A Cooperation with Donato Impedovo, Giuseppe Pirlo, Francesco Morizio Mangini, Donato Barbuzzi, Andrea Rollo, Allesandro Balestrucci, Sebastiano Impedovo, Lucia Sarcinella, Department of Computer Science, University di Bari, Italy.
- D. Impedovo, G. Pirlo,F. M. Mangini, D. Barbuzzi, A. Rollo, A. Balestrucci, S. Impedovo, L. Sarcinella, C. O'Reilly, R. Plamondon, Writing Generation Model for Health Care Neuromuscular System Investigation, Proceedings of the 10th International Meeting on Computational Intelligence Methods for Bioinformatics and Biostatistics, Nice, France, In Press.

Alzheimer Handwriting Analysis

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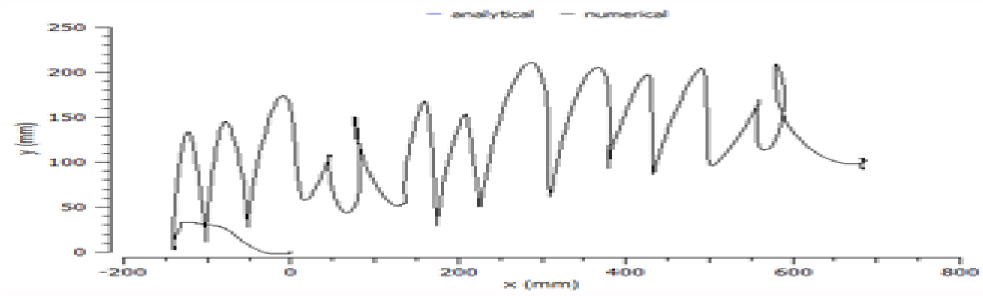
ISUNIBA DATABASE

- In order to investigate in depth the relationship between handwriting and Alzheimer disease, a detailed investigation on 50 patients has been developed by acquiring the basic words: "mamma".
- In fact the word "mamma" is one of the first learned and written by everybody; and it remains one of the last forgotten in the live.

WORD «mamma»


analytical
numerical
250
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y (mm)
-200
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600
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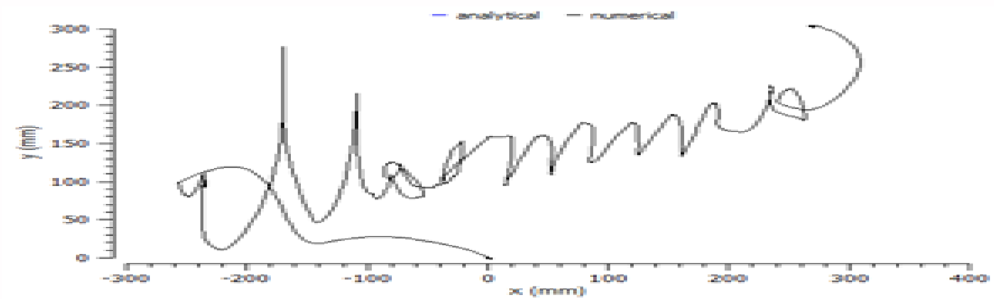
x (mm)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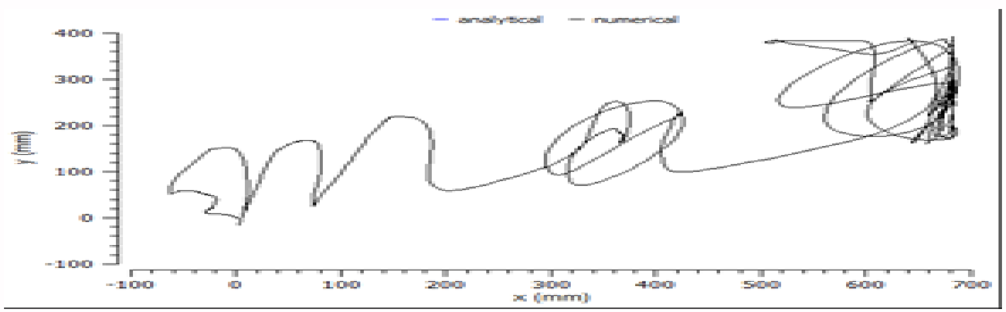
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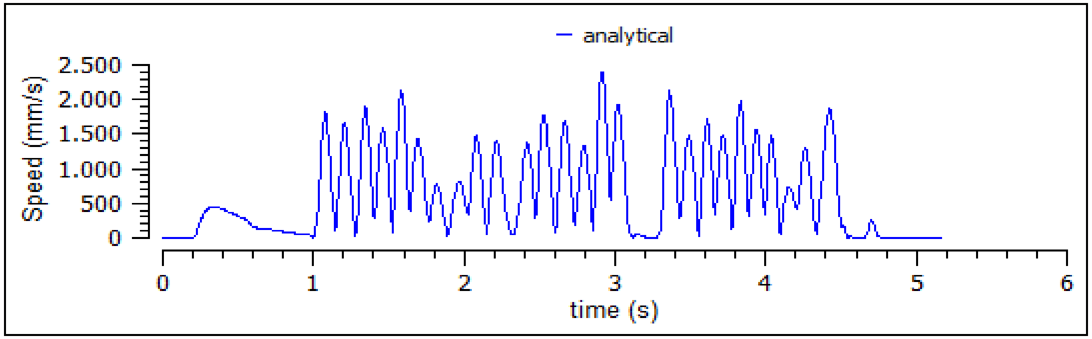
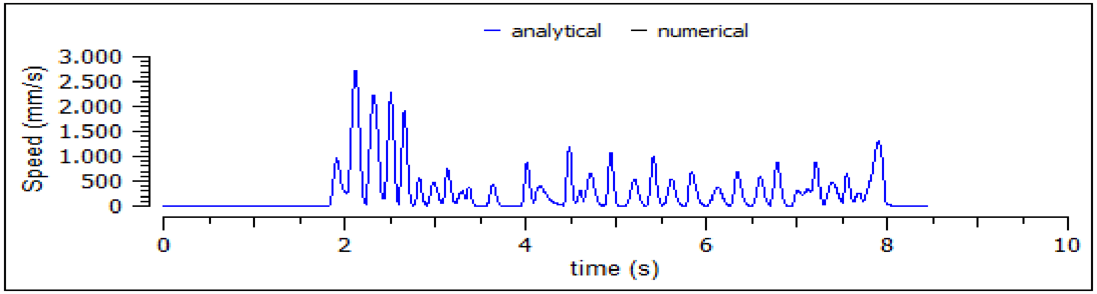
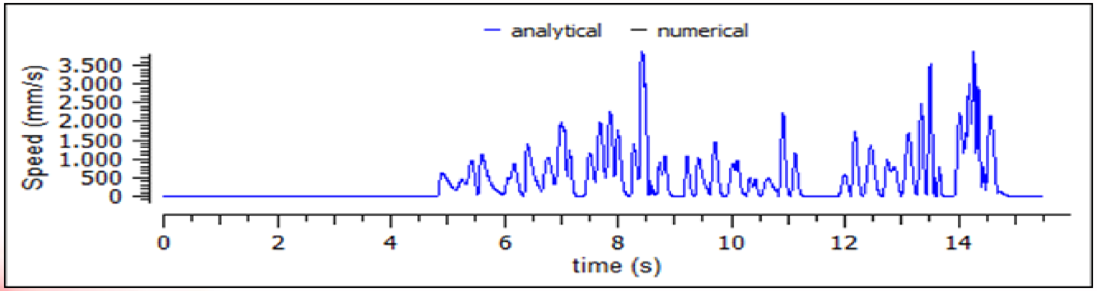
x (mm)

VELOCITY PROFILES







Synthetic Signature Generation

- A cooperation with Javier Galbally, Julian Fierrez and Javier Ortega-Garcia, Biometric Recognition Group, ATVS, Universitad Autonoma de Madrid.

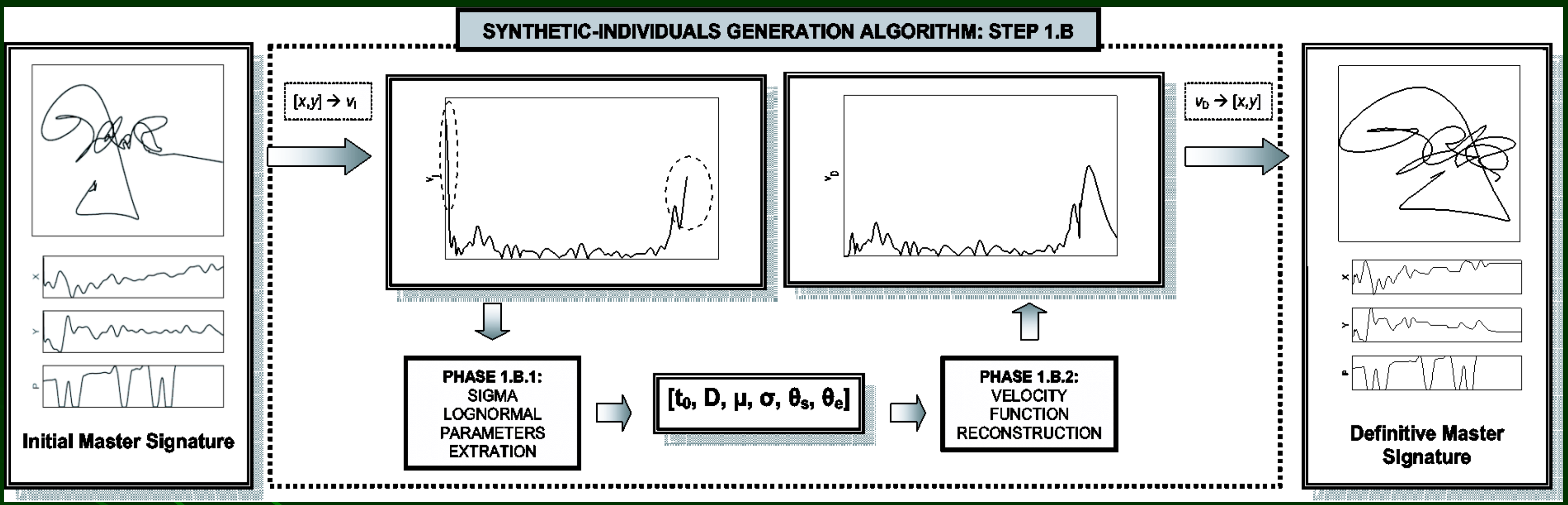
- GALBALLY, J., PLAMONDON R., FIERREZ, J. ORTEGA-GARCIA, J., "Synthetic On-Line Signature Generation Part I: Methodology and Algorithms, Pattern Recognition, Vol 45, No.7, pp. *2610-2621* 2012.
- GALBALLY, J., FIERREZ, J. ORTEGA-GARCIA, J., PLAMONDON R., " Synthetic On-Line Signature Generation Part II: Experimental Validation, Pattern Recognition, Vol.45, No.7, pp. 2622-2632, 2012.

CONTEXT

There is a need for the collection of new data that permit the objective and statistical evaluation of the performance of signature verification systems.

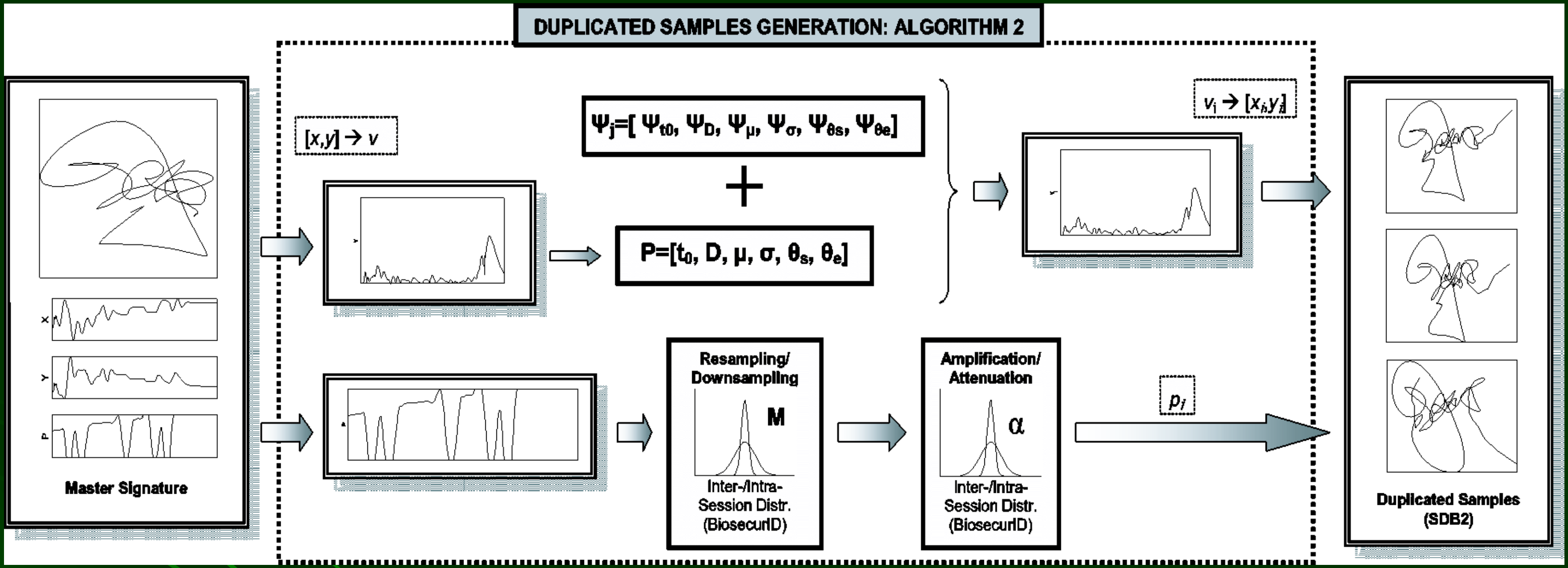
A solution: the generation of realistic synthetic signatures.

Step 1b

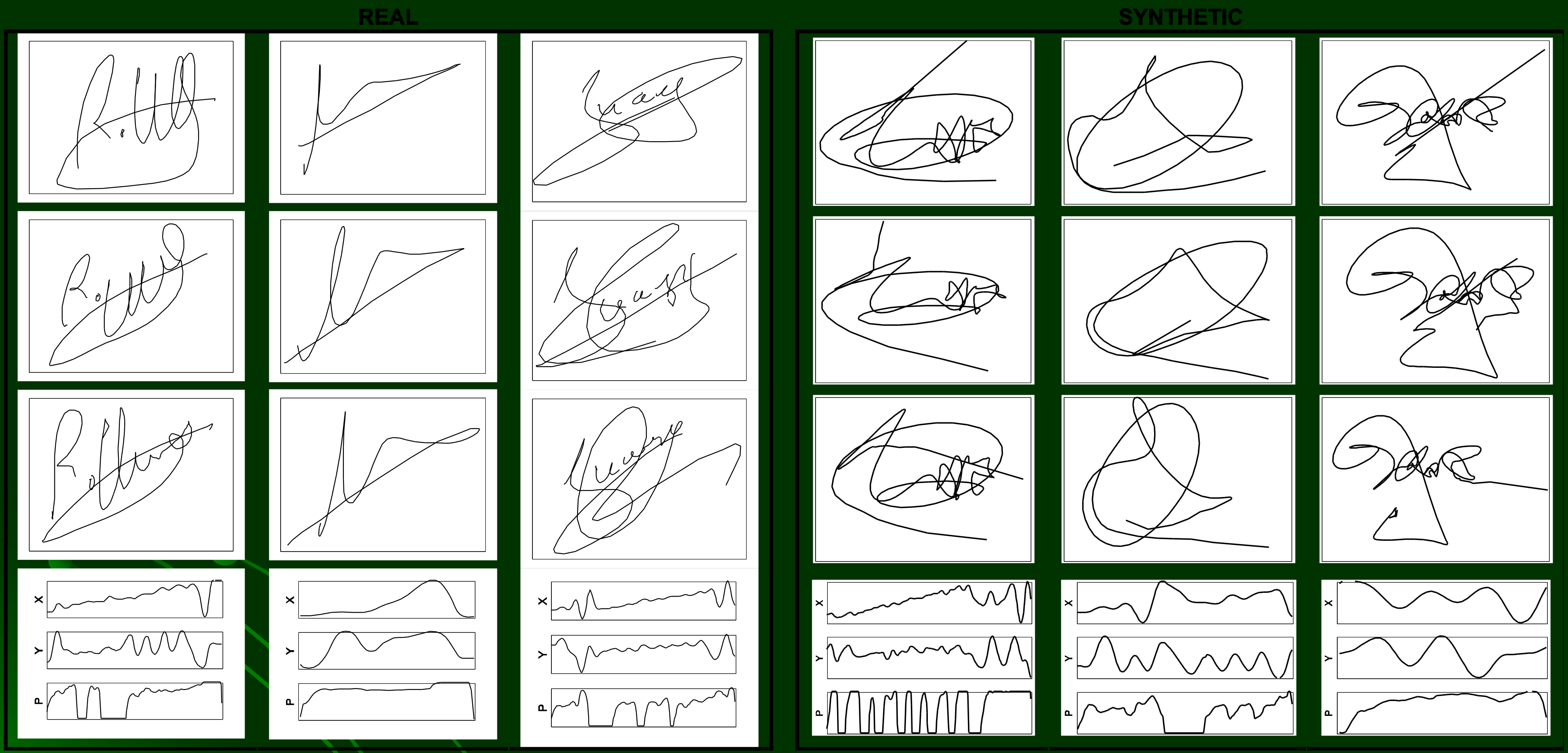


SYNTHETIC-INDIVIDUALS GENERATION ALGORITHM: STEP 1.B
Initial Master Signature
[x,y] → v_I
PHASE 1.B.1: SIGMA LOGNORMAL PARAMETERS EXTRATION
$[t_0, D, \mu, \sigma, \theta_s, \theta_e]$
PHASE 1.B.2: VELOCITY FUNCTION RECONSTRUCTION
v_D → [x,y]
Definitive Master Signature

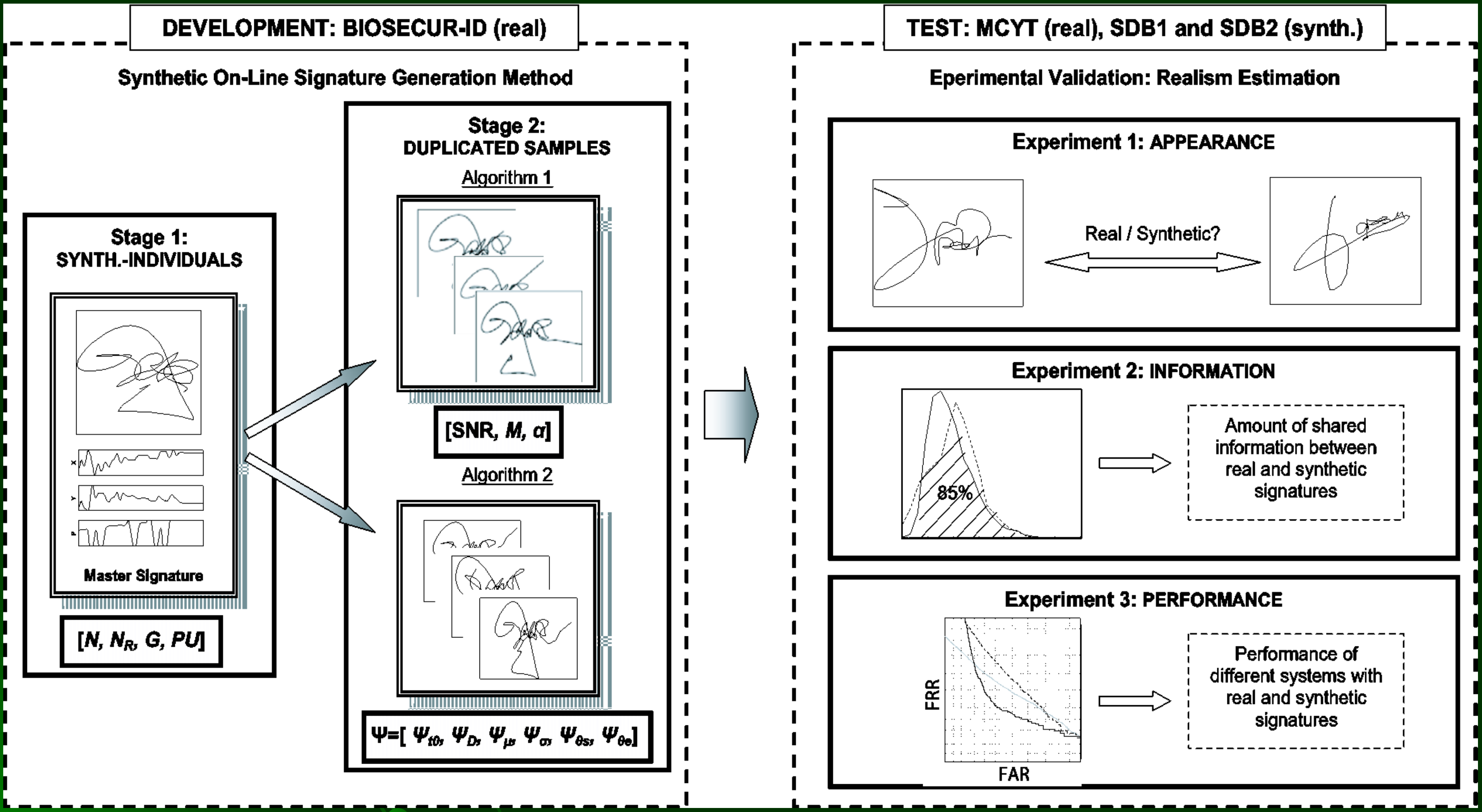
Duplicate sample generation #2



TYPICAL RESULTS



Validation strategy



Summary

The validation protocol and results described in this study work have demonstrated that, from a computer-based recognition point of view, the databases produced following the proposed generation approach are fully representative of the different real signatures that may be found in every day life in a western-European context.

Command Gesture Generation

- A Cooperation with Guy Lorette, Éric Anquetil and Abdullah Almaksour, INRIA, Université de Rennes 1, France.

- ALMAKSOUR, A., ANQUETIL, E., PLAMONDON, R., O'REILLY, C., Synthetic Handwritten Gesture Generation Using Sigma-Logormal Model for Evolving Handwriting Classifier, **Proc. 15th Biennal Conf. of the International Graphonomics Society**, (IGS 2011), Cancun, Mexico, June 12-14, 2011, pp. 98-101.

Incremental Learning of Handwritten Gesture Recognition Systems with few data

A. Almaksour E. Anquetil

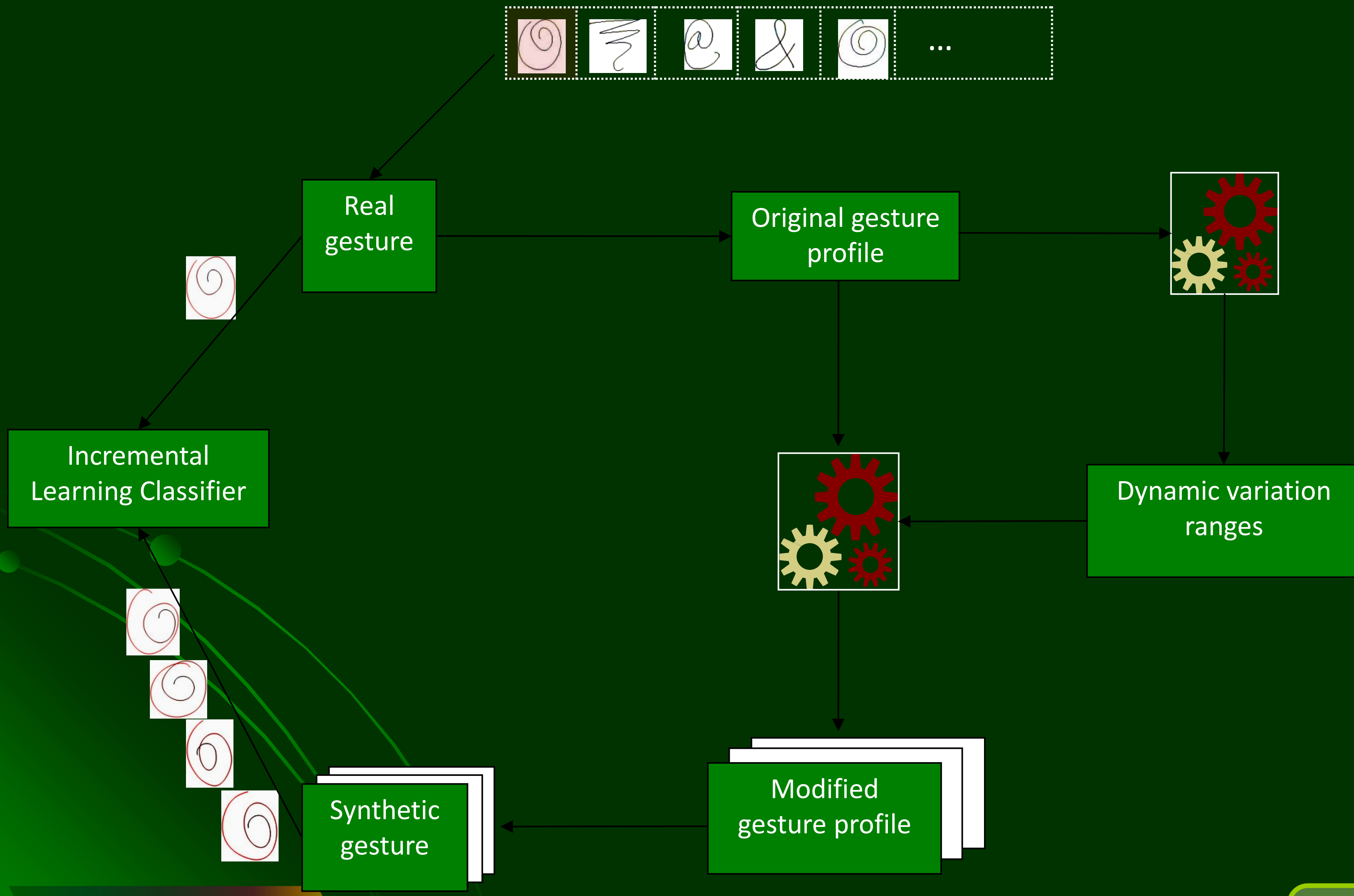
- Context: online handwriting recognition systems for pen-based interfaces (PDAs, Tablet PCs.. Etc)
- Drawbacks
  - Writer-independent recognizer → high resource cost, low accuracy
  - Pre-defined alphabet (closed vocabulary)
    - Exhaustive alphabet → high cost, low accuracy
    - Limited alphabet → can't fit new user's needs
- Proposed Solution
  - Customized recognition system
  - No pre-defined learning alphabet (learning from scratch)
  - Writer-dependent
  - **Online and incremental learning**

Learning acceleration using synthetic data

- Synthetic handwritten gesture generation techniques can be used to overcome the problem of lack of examples when a new unseen class is added.

- Synthetic data are generated by applying deformations on:
  - Lognormal profile level (much more realistic synthetic data)
    - Lognormal parameters variation within reasonable ranges
      - Predefined static variation ranges
      - Dynamic (incrementally estimated) variation ranges

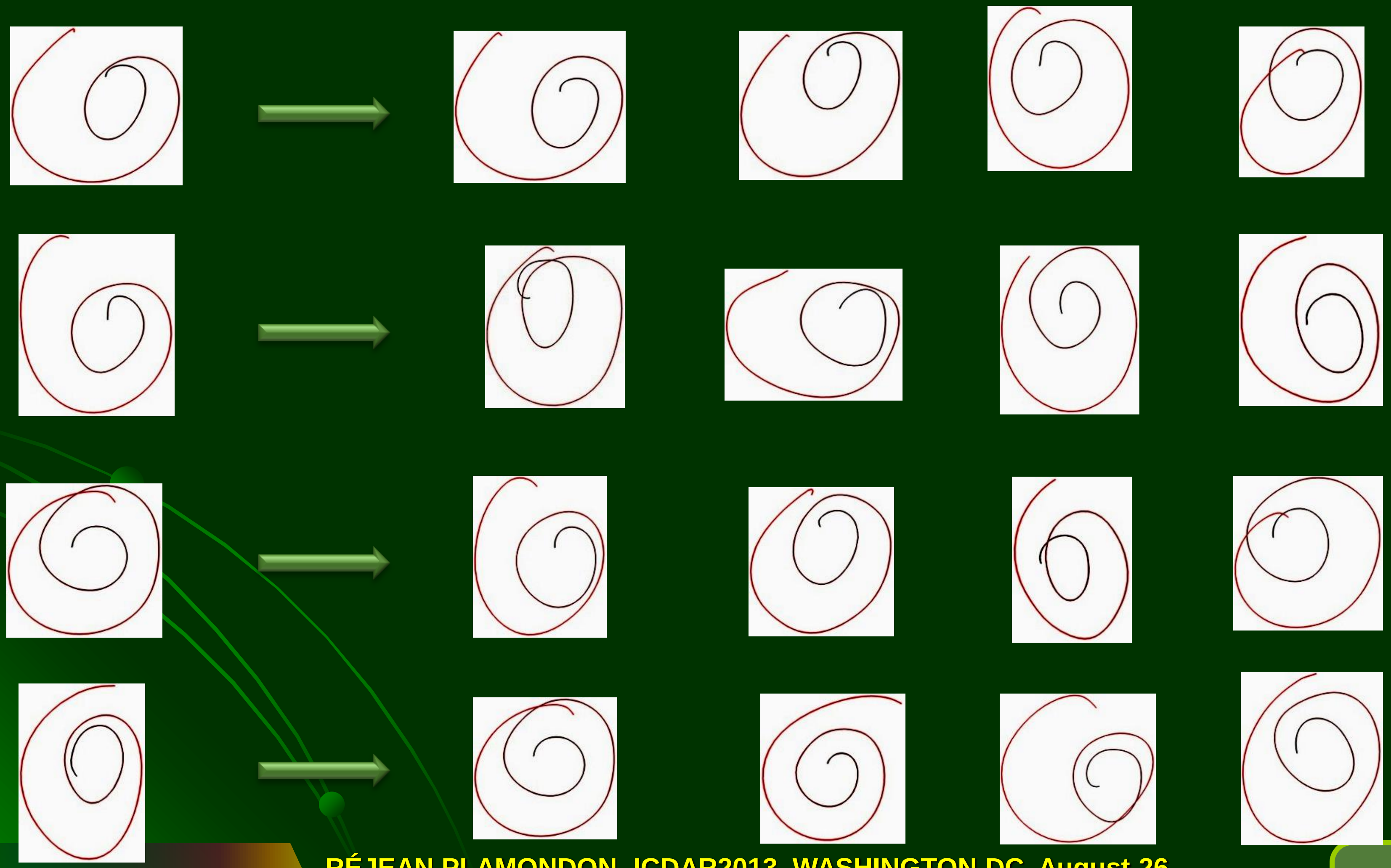
Dynamic variation ranges



Lognormal-based handwriting generation (example)

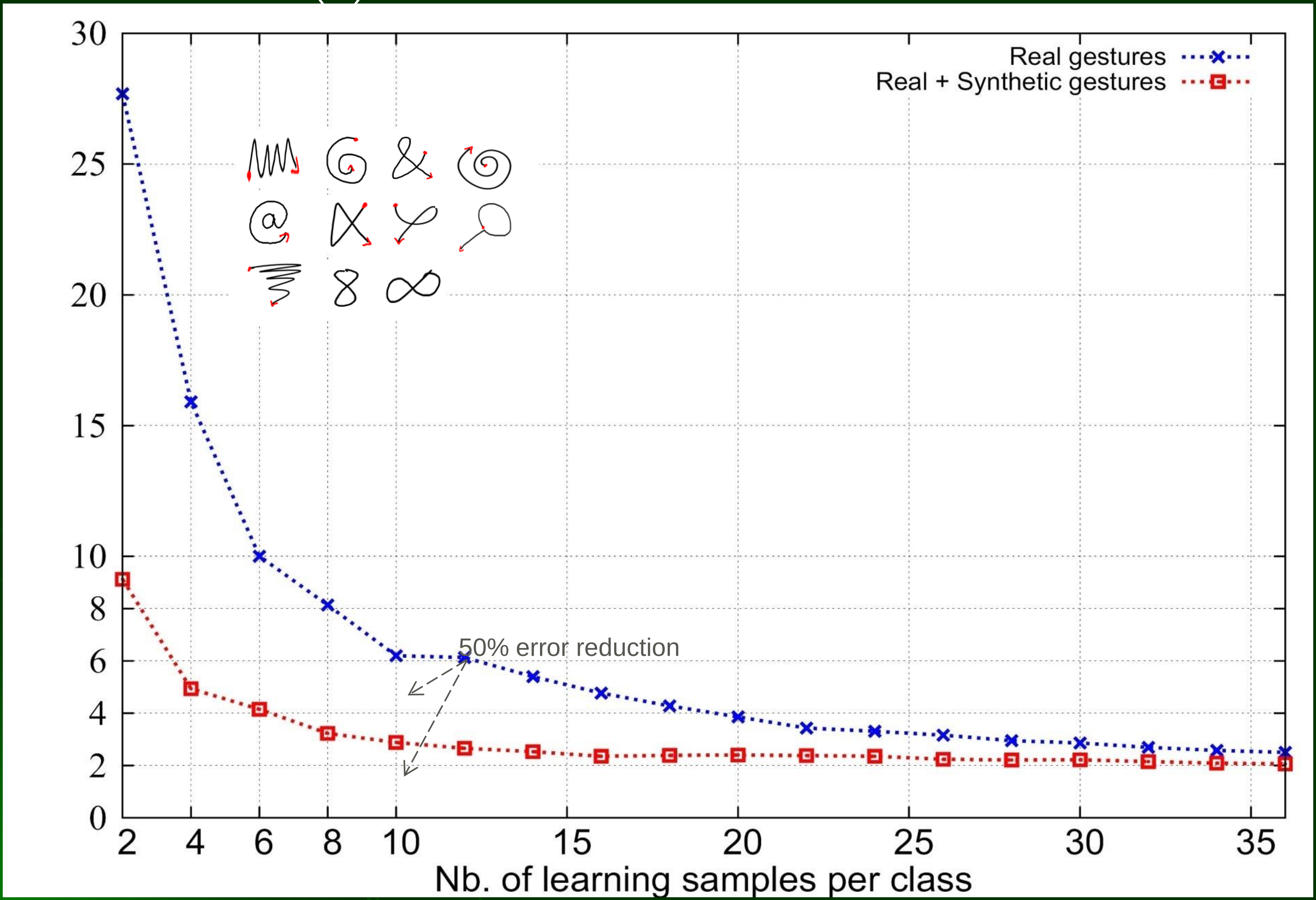
Real gestures

Synthetic gestures



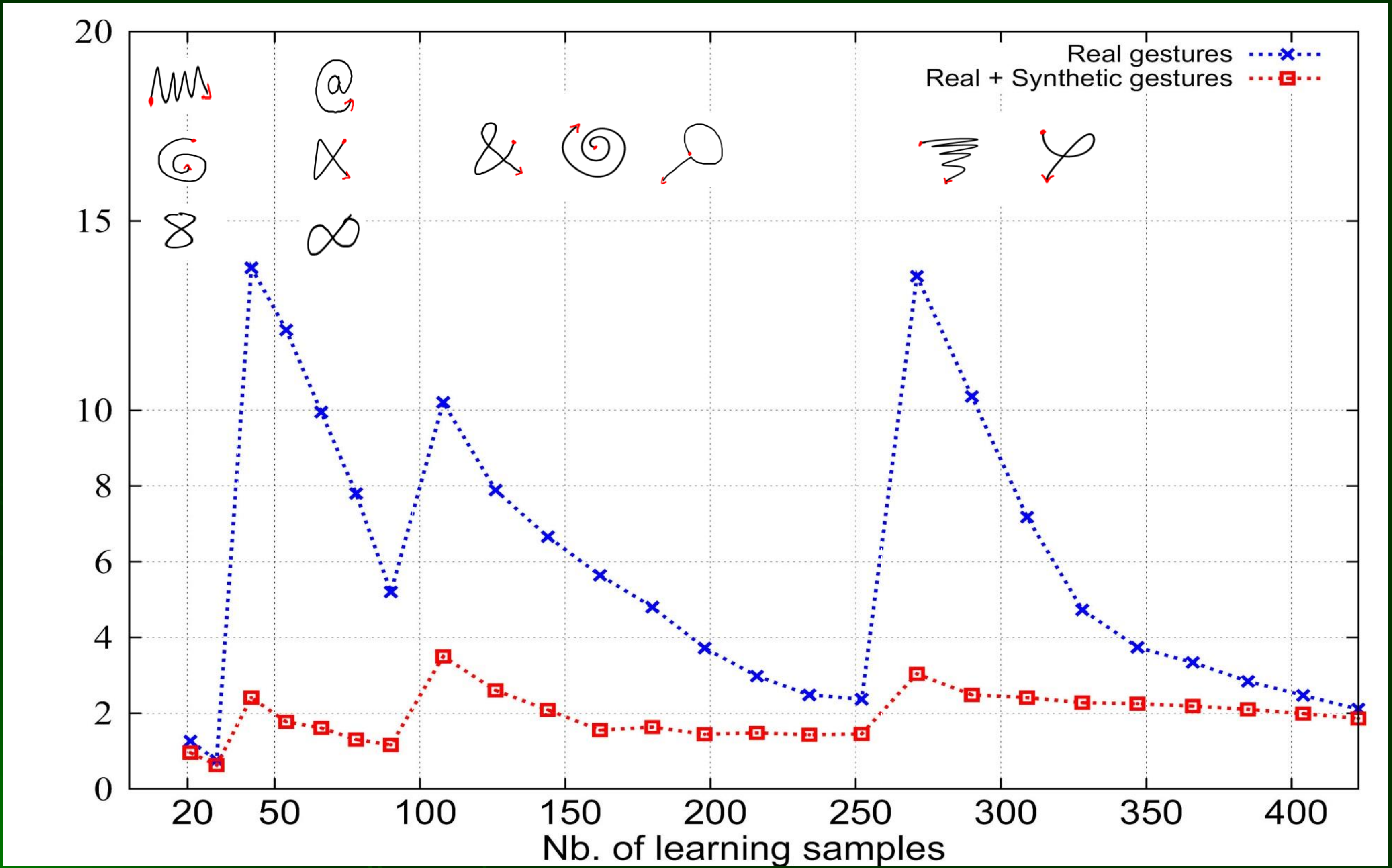
Results: Faster Learning

Misclassification rate (%)



Results: Robustness

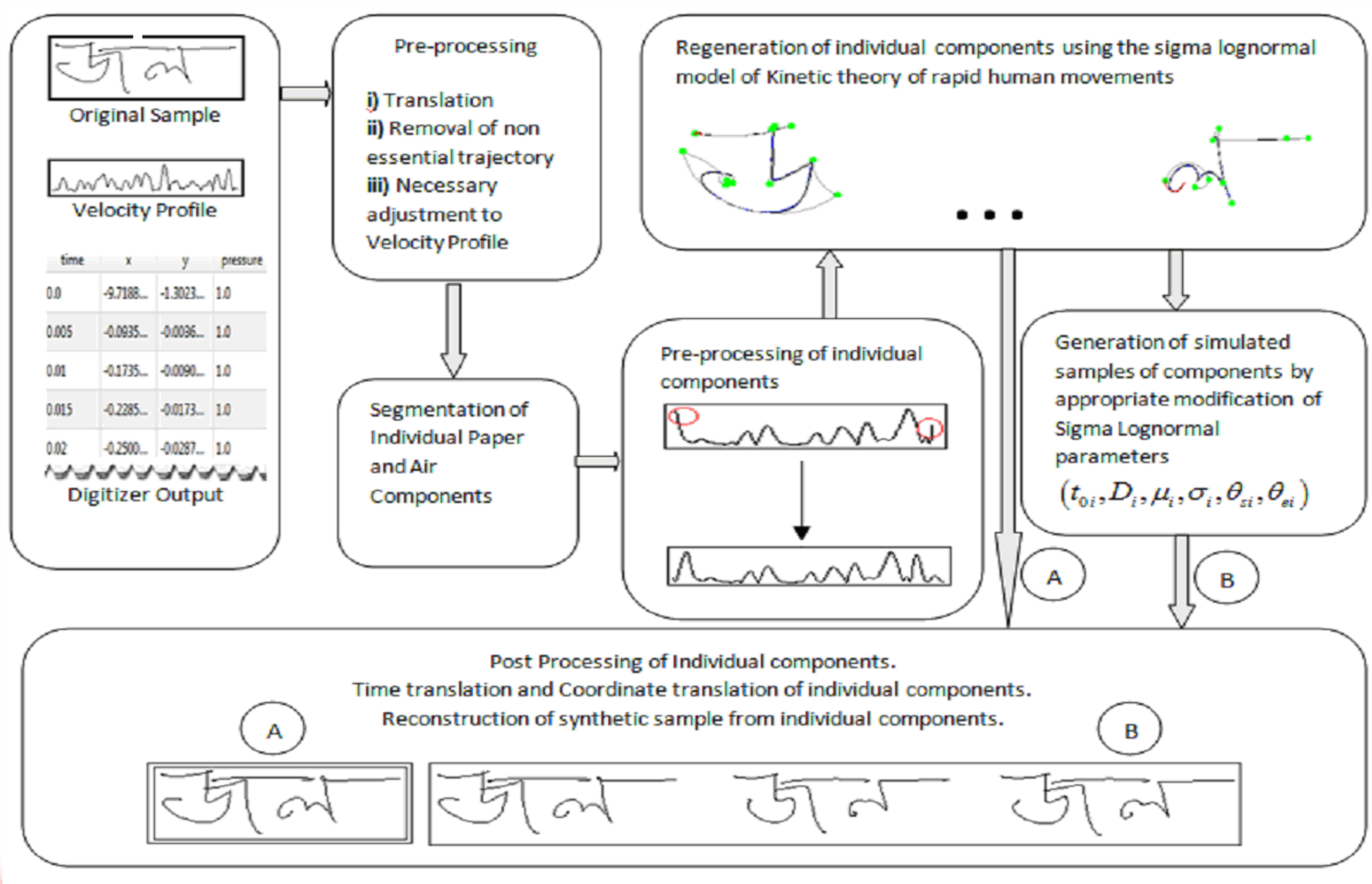
Misclassification rate (%)



Bangla Data Base Generation

- A Cooperation with Ujjwal Battacharya, Souvik Dutta, Swapan K. Parui and Pankaj Goyal, Computer Vision and Pattern Recognition Unit, Indian Statistical Institute, Kolkata, India.
- A Script and Language Independent Approach for the Generation of Huge On-line Synthetic Handwriting Databases, Submitted.

Generation Methodology



Real vs Synthetic Specimens

Preliminary Results

- A three step human evaluation has been completed so far: the results allowed us to fix the range of parameter variations that preserved word legibility and the human likeliness of the generated trajectory.
- Computer evaluation under way: we are using the synthetic samples in various on-line handwriting recognition environments.

Captcha Generation

- A Cooperation with Venu Govindaraju and Chetan Ramaiah, Deparment of Computer Science and Engineering, Center for Unified Biometric and Sensors, SUNY at Buffalo, USA.

- RAMAIAH, C., PLAMONDON, R., GOVINDARAJU, V., Handwritten CAPTCHA generation based on the Sigma-Lognormal model, Proc.16th Biennial Conf. of the Graphonomics Society, Nara, Japan, June 10-14, 2013, pp. 105-108.

Sigma-Lognormal model for handwritten CAPTCHAs

- Randomly pick out a word from a online handwriting database.
- Determine the parameters of the Sigma-Lognormal model for the chosen word.
- Modify the parameters of the model within an acceptable range of values.
- Render the modified word as the handwritten CAPTCHA.

Experiments

- Experiment 2A results:

0.5 0.7 1 1.3 1.5

A sample where the parameters were modified by the same quantity. The numbers below the images indicate the ratio between the original values and the modified values.

Experiments

- Experiment 2A results

Ratio	0.5	0.6	0.7	0.8	0.9	1.1	1.2	1.3	1.4	1.5
WMR accuracy	5.52	13.77	18.92	17.89	33.36	29.23	16.86	20.98	5.52	3.72
Human Evaluation	48.45	70.1	87.62	96.9	100	100	97.93	87.62	64.94	50.51

- The ratio indicates how much each parameter was changed with respect to its original value.
- A range of 0.7-1.3 might be appropriate for our purposes.

Conclusions

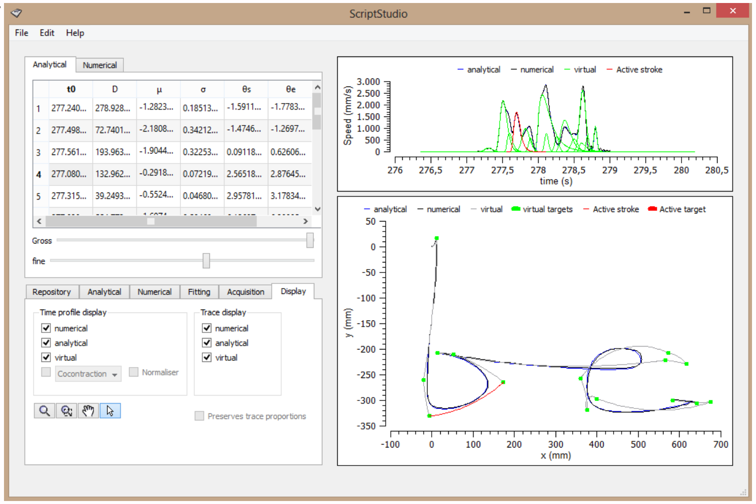
- The sigma-lognormal model is an efficient representation of handwriting.
- CAPTCHAs generated via this technique can prove to be a viable alternative to existing methods.
- In addition to the current approach, adding noise and distortion can greatly improve efficacy of the CAPTCHAs.

A Pen Based Interface

- A Cooperation with Josep Llados and Alicia Fornes, Computer Vision Center, Universitad Autonomous de Barcelona, Spain.
- Pen-based user-centred tool for on-line word spotting and fine motricity monitoring to evaluate diseases in clinical research, to design rehabilitation task, to implement learning interfaces.

Analysis of Online Handwriting

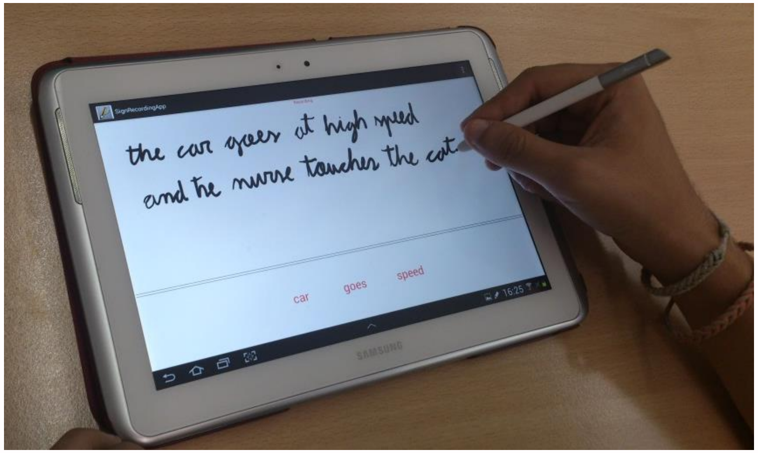
- Online Handwriting is related to neurology
  - Detection of early stages of diseases
    - Parkinson, Brain Strokes, Alzheimer…
  - Sketching for Rehabilitation (accidents)
    - Monitoring the movements' evolution
  - Learning
    - Children (dyslexia, attention, …)
  - Detection of stress, depression



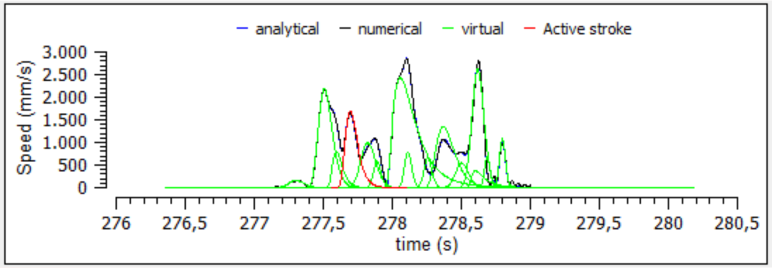
- Online interfaces
  - Devices:
    - Tablets
    - Samsung Galaxy Note series

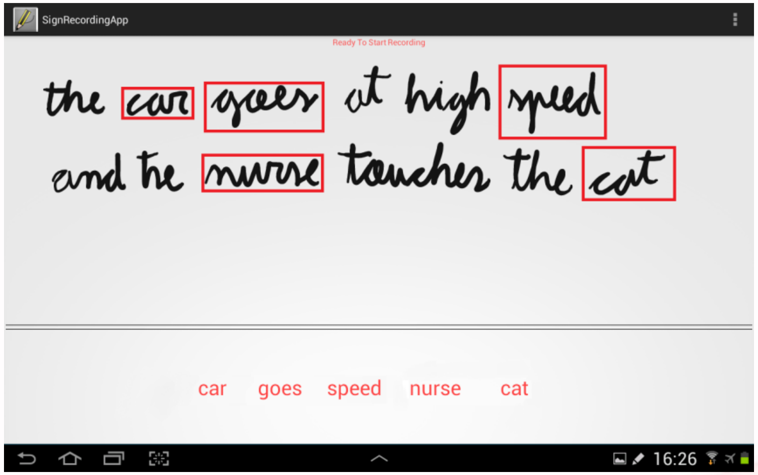
Proposed System: A Pen-based Interface for On-line Word Spotting and Fine Motricity Monitoring

1. **On-line word spotting**
   - The user defines his key-words
   - The system recognizes the key-words in the sentences written on the tablet



2. **Kinematic Analysis**
   - For each detected key-word, it computes the kinematic parameters and compares with the ones from stored instances of the same key-word:
     Motricity monitoring (evolution of HW)





5 – The tip of an iceberg

There Is Much More to Do with LOGNORMALITY!!!

Potential Applications

- On-line handwriting recognition / Signature Verification:
- a new representation space, automatic segmentation…
- writer style characterization, automatic data base generation
- new on-line recognizers and verifiers
- interactive tools to help children to learn handwriting...
- Biomedical signal processing:
- a new set of parameters to characterize the human motor control system…
- design of psychomotor evaluation tests
- detection of fine motor control problems (Parkinson, Alzheimer, CVA)
- prevention and rehabilitation tests and tools
- effects of medication, alcohol, drugs, weight loss…
- New open fields:
- a new set of functions for 2D and 3D smoothest curve modeling
- anthropomorphic arm design
- exoskeletons and prosthetics
- humanoid movements modeling of virtual reality objects.

Other agreements under discussion

- Cooperation with Thierry Artières, Université Pierre et Marie Curie (Paris 6), France.
- Cooperation with Angelo Marcelli and Rosa Senatore, Univertity di Salerno, Italy.
- Cooperation with Sonia Garcia, Institut Mines-Télécom/Télécom SudParis, France.
- Cooperation with Miguel Angel Ferrer Ballester, Universitdad de Las Palmas de Gran Canaria, Spain.
- Cooperation with Christophe Champod, Université de Lauzanne, Switzerland.
- Cooperation with Joanna Putz-Leszczynska, Politechnika Warszawska, Poland.

Any group interested ?

TWO TYPES OF AGREEMENTS

Share our knowledge and software with research partners

$hare our knowledge and software with commercial partners

Back to Poetry and Geometry...

Searching for the Underlying Thruths?

Existential Geometry

The straight line hesitates

The circle questions itself

The sphere wallows
in the symmetry
of beautiful
tautological answers

Time cares nothing
like a fetus

Géométrie Existentielle
translated by
Andrea Zanin.

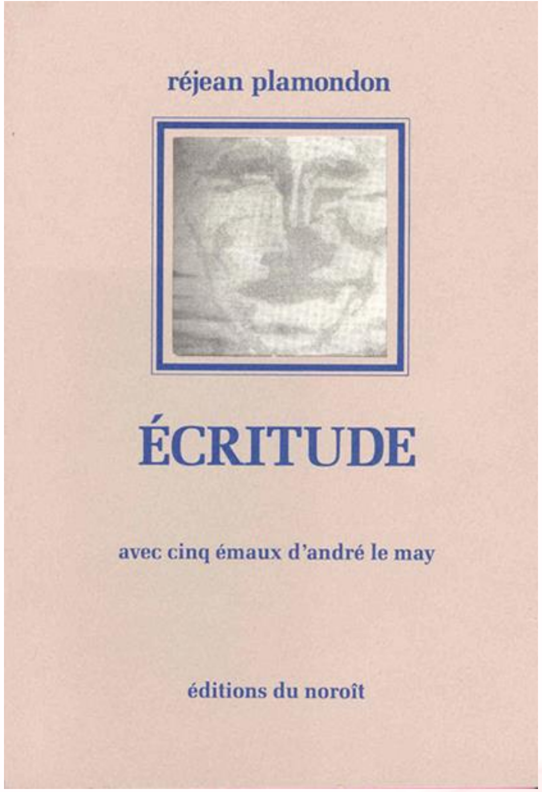
Géométrie Existentielle

la droite hésite

le cercle s’interroge

la sphère se vautre
dans la symétrie
des belles réponses
tautologiques

le temps s’en fout
comme un fœtus



A PRIVATELY AND SOLITARY ADVENTURE

With the support of my son
Claudéric Ouellet-Plamondon
B.Sc.A, M.Sca., Physics engineering
for the various computer simulations
and figures preparation.

APPLYING PATTERN RECOGNITION TECHNIQUES TO PROVIDE SOME NEW INSIGHTS TO THE UNIFICATION OF PHYSICS

PLAMONDON, R., **Patterns in Physics: Toward a Unifying Theory,** Presses Internationales Polytechnique, Montréal, June 2012, 214 pages, ISBN 978-2-553-01633-2 ,
http://www.presses-polytechnique.ca/en/patterns-in-physics

Fasten your seat belt!

A Crash Course on General Relativity and Quantum Mechanics

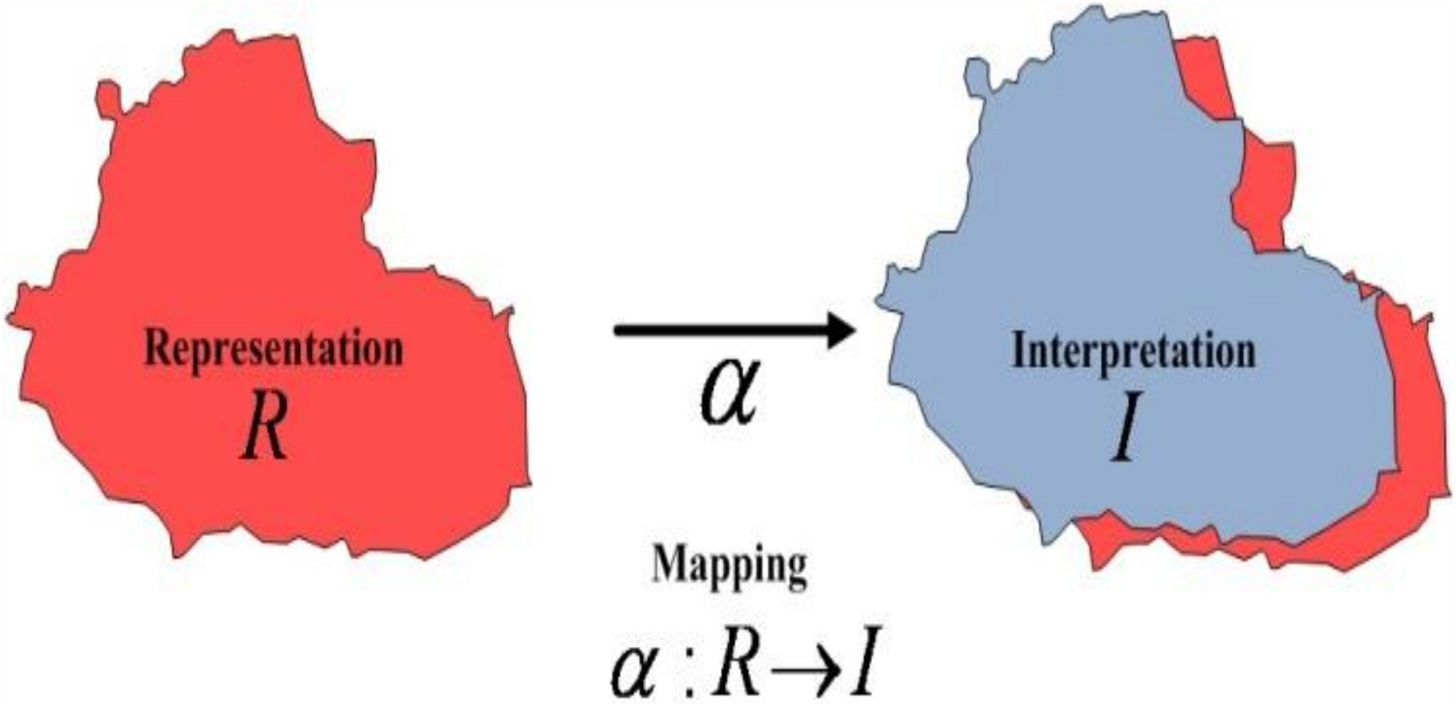
PLAMONDON, R.’ O’REILLY, C,. OUELLET-PLAMONDON, R., Strokes against Strokes-Strokes for Strides, In Press, Pattern Recognition, 2013, availlable online at http://authors.elsevier.com/sd/article/S0031320313002082

Part 2:TOPICS

1. A statistical pattern recognition approach
2. Putting general relativity into a probabilistic context (The interdependence principle)
3. Estimating the probability density
4. A symmetric geometry
5. An axisymmetric geometry
6. Three supplementary emerging interactions
7. From stars to galaxies… to the Universe
8. Take home messages

Statistical Pattern Recognition

Simon, J.C., (1984), *La reconnaissance des formes par algorithmes.* Masson, Paris.



Patterns are generated by a probabilistic system

Statistical Pattern Recognition

- REPRESENTATION

  arbitrary scaled features ⇔ N-dimensional space
  object⇔ random vector

- INTERPRETATION

  class ⇔ a cluster defined by a density function

- MAPPING

  class delimitation ⇔ discriminating function

TOPICS

1. A statistical pattern recognition approach
2. Putting general relativity into a probabilistic context (The interdependence principle)
3. Estimating the probability density
4. A symmetric geometry
5. An axisymmetric geometry
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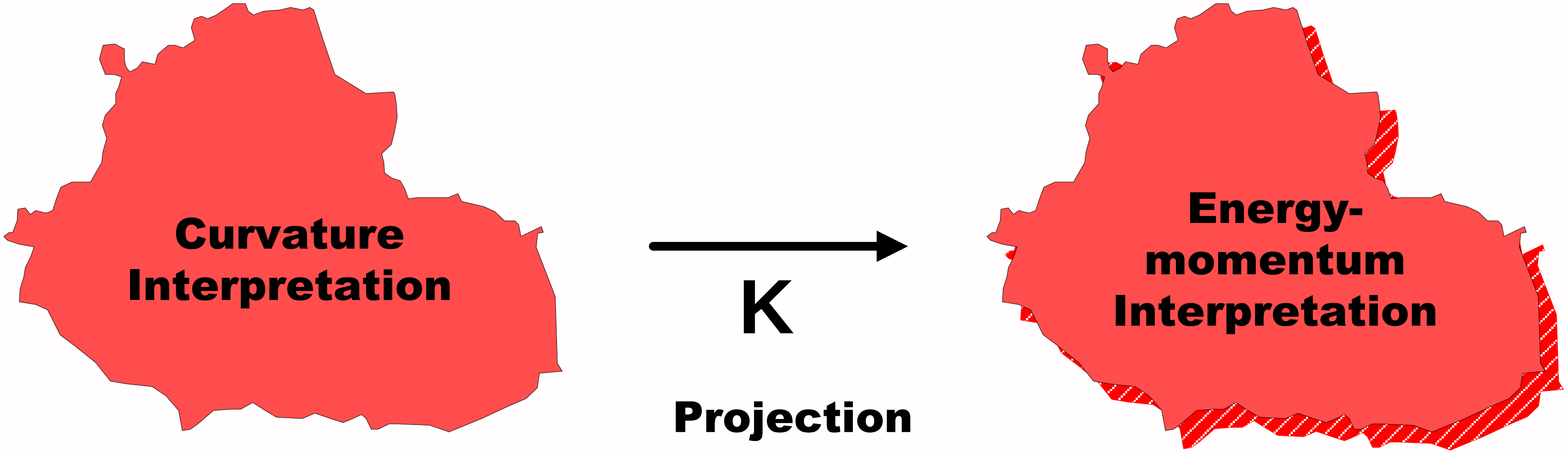
7. From stars to galaxies… to the Universe
8. Take home messages

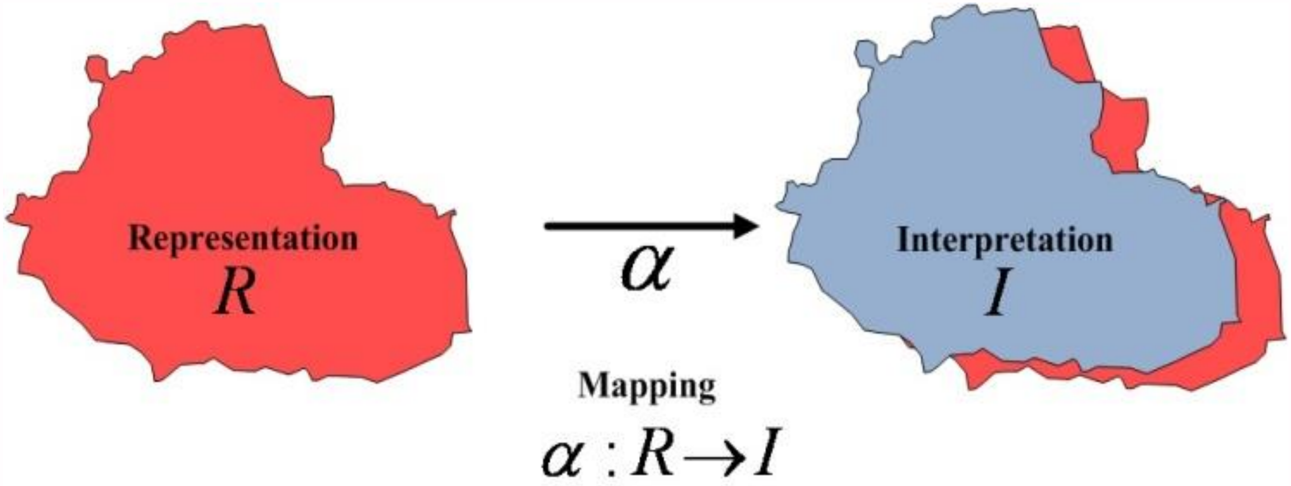
Einstein's equation

$$G = KT$$



A Relationship Between Two Interpretation Spaces

Statistical Pattern Recognition



Two information spaces must be analyzed and compared.

First information space: the structure of space-time

- REPRESENTATION
  - arbitrary coordinates +metrics ⇔ 4-dimensional space
  - metric quantified coordinate ⇔ arbitrary features of the manifold
- MAPPING
  - Einstein tensor: G
- INTERPRETATION
  - 16 component curvature space

Second information space: the content of space-time

- REPRESENTATION
  - arbitrary coordinates +metrics ⇔ 4-dimensional space
  - metric quantified coordinates ⇔ localization of events
- MAPPING
  - Momentum-Energy tensor: T
- INTERPRETATION
  - Mass-energy density, energy flux, momentum density and stress components

Einstein's equation

$$G = KT$$

- The Einstein's equation can be seen as making a link between the two interpretation spaces.
- BUT, according to the statistical pattern recognition paradigm, these interpretation spaces could be given a probabilistic meaning...How?

Part 2: TOPICS

1. A statistical pattern recognition approach
2. Putting general relativity into a probabilistic context (The interdependence principle)
3. Estimating the probability density
4. A symmetric geometry
5. An axisymmetric geometry
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8. Take home messages

Interdependence principle

Spacetime curvature (S) and matter-energy density (E) are two inextricable information spaces defining the physically observable probabilistic universe (U); they must be mutually exploited to describe any subset U_i *of this universe. In terms of expectations, the probability of observing a subset* (U_i) *is:*

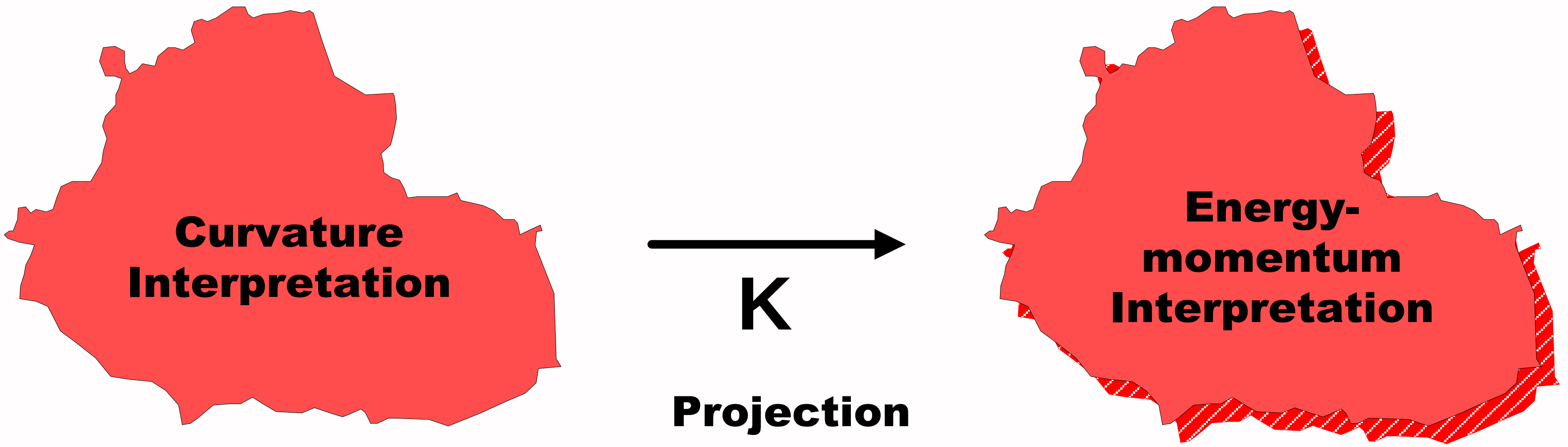
$$P(U_i) = P(S_i, E_i) < 1$$

Corrolary

The probability of observing and describing a given subset of the universe P(U_i), that is the joint probability of P(S_i,E_i), can be studied from two equivalent *modi operandi*: either by analyzing the structure of the spacetime as an interpretation space associated with an *a priori* given matter-energy distribution or by analyzing the matter-energy density as an interpretation space associated with an *a priori* given spacetime structure.

In terms of Bayes'law...

(conditionnal probabilities)



$$P(U_i) = P(S_i, E_i) = P(S_i / E_i) P(E_i) = P(\mathrm{E}_i / \mathrm{S}_i) P(S_i)$$

$$f(S_i / E_i) f(E_i) = f(\mathrm{E}_i / \mathrm{S}_i) f(S_i)$$

$$f(S_i / E_i) = f(\mathrm{E}_i / \mathrm{S}_i) \frac{f(S_i)}{f(E_i)}$$

A link with Einstein's law?

$$f(S_i/E_i) = f(\mathrm{E}_i/\mathrm{S}_i)\frac{f(S_i)}{f(E_i)} \Leftrightarrow G = KT$$

$$f(S_i/E_i) = k_1 trG$$

$$\frac{f(S_i)}{f(E_i)} = k_2 trT$$

$$f(\mathrm{E}_i/\mathrm{S}_i) = ?$$

Part 2: TOPICS

1. A statistical pattern recognition approach
2. Putting general relativity into a probabilistic context (The interdependence principle)
3. Incorporating quantum mechanics
4. A symmetric geometry
5. An axisymmetric geometry
6. Three supplementary emerging interactions
7. From stars to galaxies… to the Universe
8. Take home messages

A Potential Pathway…

$f(E_i \ / \ S_i)?$ Reflects the probability of presence of a given energy momentum density

In Quantum Mechanics, the wave function ψ_{E_i} can be used to compute the probability of presence of a given particle

$$\psi^*_{E_i}\psi_{E_i} = f_\psi(S_i)$$

Estimating the probability of presence

- Building a star from scratch by adding numerous identical particles ($N \rightarrow \infty$), each one with its own wave function, density function and associated space-time, as seen from a locally flat tangent space.
- Making the convolution of their corresponding density functions.
- The central limit theorem predicts that the ideal form of the global probability density $f(\boldsymbol{x})$ will be a Gaussian multivariate in a flat space-time.

In a Curved Space-time…

- The Gaussian multivariate becomes a « **Curved Gaussian** »:

$$f(\bar{r}) = \frac{1}{4\pi^2\sigma^4}\exp\left(-\frac{\bar{r}^2}{2\sigma^2}\right) \Rightarrow \text{ External Flat Space}$$

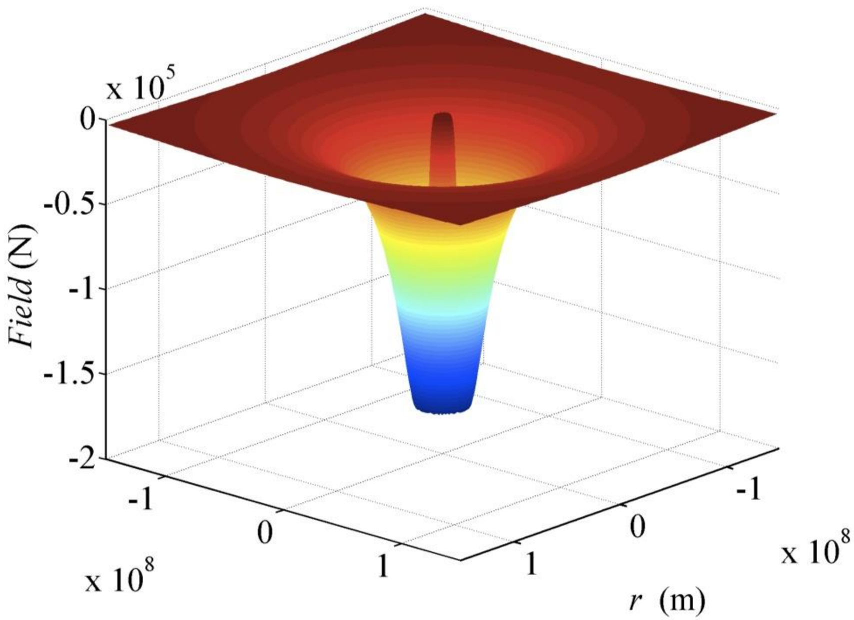
$$\text{Mapping} \Rightarrow \frac{x^2}{\sigma^2} = \frac{\sigma^2}{r^2}$$

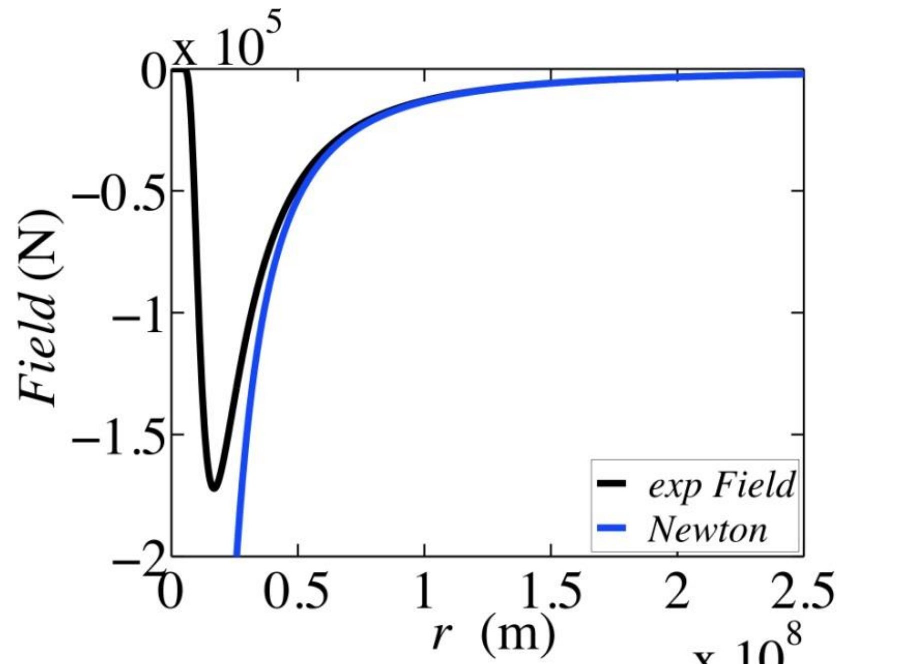
$$f(E_i / S_i) = k_3 f \;\; \widehat{r} \;\; = \frac{1}{4\pi^2\sigma^2\widehat{r}^2}\exp\left(\frac{-\sigma^2}{2\widehat{r}^2}\right) \Rightarrow \text{ Internal Curved Space}$$

Emergence of Newton's law of gravitation: the field

$$g(r) = -\left|\vec{\nabla}\Phi \;\; r\right| = -\frac{2KMc^4}{4\pi\sigma^{3} r^2}\exp\left(-\frac{\sigma^2}{2r^2}\right)$$

$$g(r) \cong -\frac{2KMc^4}{(4\pi\sigma)^3 r^2} = -\frac{GM}{r^2}$$

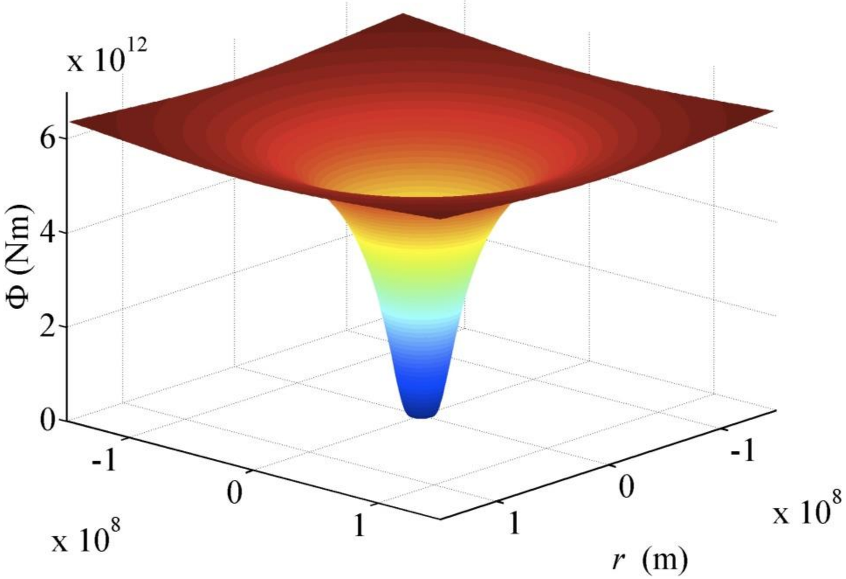


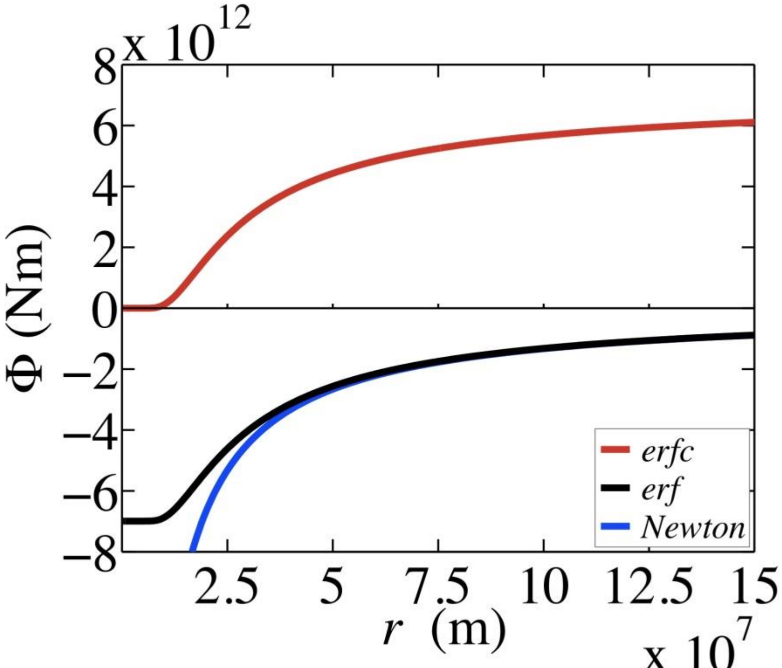


Emergence of Newton's law of gravitation: the potential

$$\Phi_{erfc}(r) = \frac{2KMc^4}{(4\pi\sigma)^3}\left(\frac{\sqrt{\pi}}{\sqrt{2}\sigma}\right) erfc\left(\frac{\sigma}{\sqrt{2}r}\right) = \Phi_{erfc}(r)$$

$$\Phi_{erf}(r) = -\frac{2KMc^4}{(4\pi\sigma)^3}\left(\frac{1}{r} - \frac{1}{6r^3} + \frac{1}{40r^5} - \ldots\right) \cong -\frac{GM}{r}$$





Brief pause

- According to the present pattern recognition paradigm, the Newton's law is not empirical. It is an approximation of a more general law. It can be seen as an emerging phenomenon when the proper representation and interpretation spaces are used to describe a physical manifold.
- The resulting *erfc* potential can be incorporated in a metric to study statically symmetric system, following Einstein's methodology.

Part 2: TOPICS

1. A statistical pattern recognition approach
2. Putting general relativity into a probabilistic context (The interdependence principle)
3. Estimating the probability density
4. A symmetric geometry
5. An axisymmetric geometry
6. Three supplementary emerging interactions
7. From stars to galaxies… to the Universe
8. Take home messages

The symmetric metric and the field equation

$$ds^2 = \left[1 + \frac{2}{c^2} GM \left(\frac{\sqrt{\pi}}{\sigma\sqrt{2}}\right) erfc\left(\frac{\sigma}{\sqrt{2}r}\right)\right] c^2 dt^2$$

$$- \left[1 + \frac{2}{c^2} GM \left(\frac{\sqrt{\pi}}{\sigma\sqrt{2}}\right) erfc\left(\frac{\sigma}{\sqrt{2}r}\right)\right]^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$

- No coordinate singularity
- No intrinsic singularity
- Temporal offset at infinity
- Radial delays

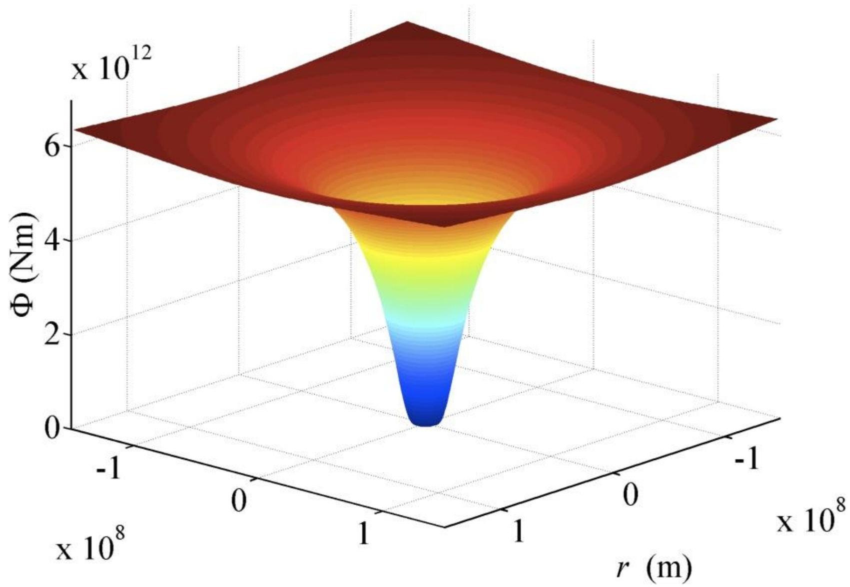
Most striking properties

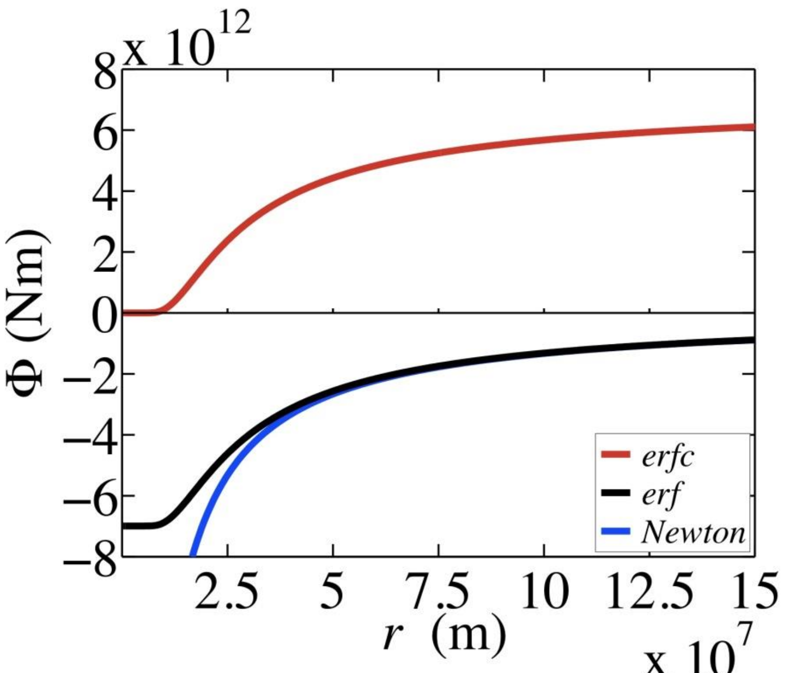
- New set of exact analytical solutions
- Converge towards Einstein's predictions at large distance
- Differs from Einstein's predictions at small distance
- There will be no gravitational collapse in systems described by such a metric
- Black holes without any intrinsic singularity
- Gauge dependent?

Emergence of Newton's law of gravitation: the potential

$$\Phi_{erfc}(r) = \frac{2KMc^4}{(4\pi\sigma)^3}\left(\frac{\sqrt{\pi}}{\sqrt{2}\sigma}\right) erfc\left(\frac{\sigma}{\sqrt{2}r}\right) = \Phi_{erfc}(r)$$

$$\Phi_{erf}(r) = -\frac{2KMc^4}{(4\pi\sigma)^3}\left(\frac{1}{r} - \frac{1}{6r^3} + \frac{1}{40r^5} - \ldots\right) \cong -\frac{GM}{r}$$





erfc vs *erf* functions

$$erfc(z) = 1 - erf(z)$$

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Two new axisymmetric components

- A rotation term:

$$\omega_{st} = \frac{d\phi}{dt} \Rightarrow +\frac{2K}{\omega_{st}} d\phi dt$$

- An expansion term:

$$v_{st} = \frac{dr}{dt} \Rightarrow +\frac{2Kv_{st}}{c^2}\left[1 - \frac{2K}{c^2} erf\left(\frac{u}{4\pi\sqrt{2r}}\right)\right]^{-1}$$

Very Brief Pause…

The axisymmetric metric can be seen as explaining why any massive body in the universe is rotating and its associated space-time looks like expanding!

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Back to the Central Limit Theorem

When the number of functions that are convolved is not infinite…

A convergence error will emerge.

This error will have three components.

Central Limit Theorem: convergence error

$$E_r(y) = f(y) - N(y) = \frac{\mu_3}{6\sigma^3\sqrt{n}}\left(\frac{y^3}{\sigma^3} - \frac{3y}{\sigma}\right)N(y) + O\left(\frac{1}{\sqrt{n}}\right)$$

$$\text{Mapping} \Rightarrow \frac{y}{\sigma} \rightarrow \frac{\sqrt{2}x}{\sigma} \Rightarrow \frac{x}{\sigma} = \frac{\sigma}{\sqrt{2}r}$$

$$\left(\frac{y^3}{\sigma^3} - \frac{3y}{\sigma}\right) \Rightarrow \left(\frac{x^3}{2\sqrt{2}\sigma^3} - \frac{3x}{\sqrt{2}\sigma}\right)$$

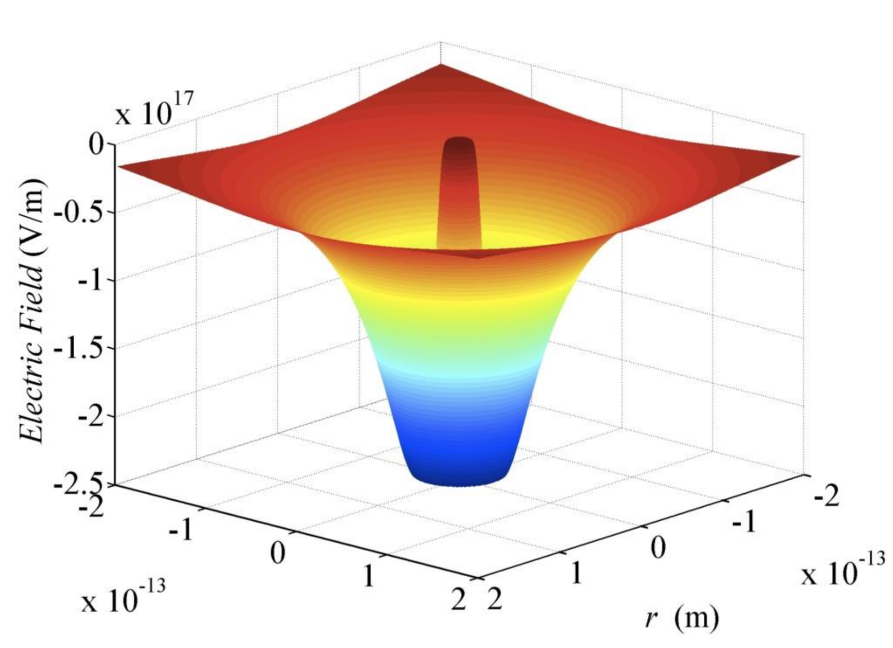
$$\nabla^2\Phi = \frac{2K_\sigma Mc^4}{4\pi^3\sigma r^5}\left[1 - \frac{\mu_3}{2\sqrt{2n}\sigma^2 r} + \frac{\mu_3}{12\sqrt{2n}r^3}\right]\exp\left(\frac{-\sigma^2}{2r^2}\right)$$

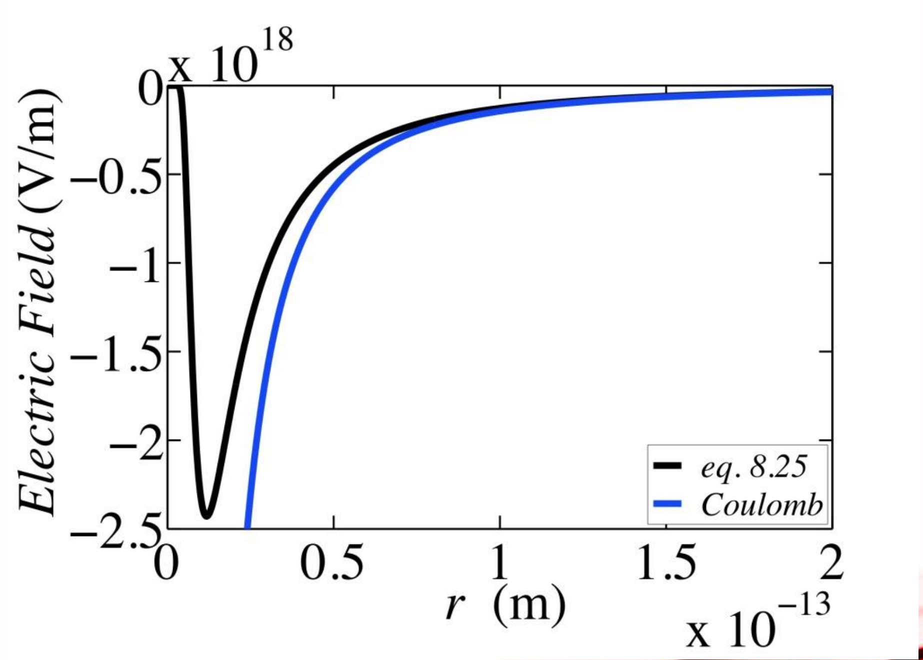
Emergence of Coulomb's field

$$\mu_3 = \frac{8\pi c_2 Q^2}{m_{ref} u \varepsilon_0^2}$$

$$g_e(r) = \frac{\mu_3 u \varepsilon_0}{32\pi^2 c_2 r^2} erfc\left(\frac{c_3 \varepsilon_0}{64\pi^2 c_2 \sqrt{2} r}\right)$$

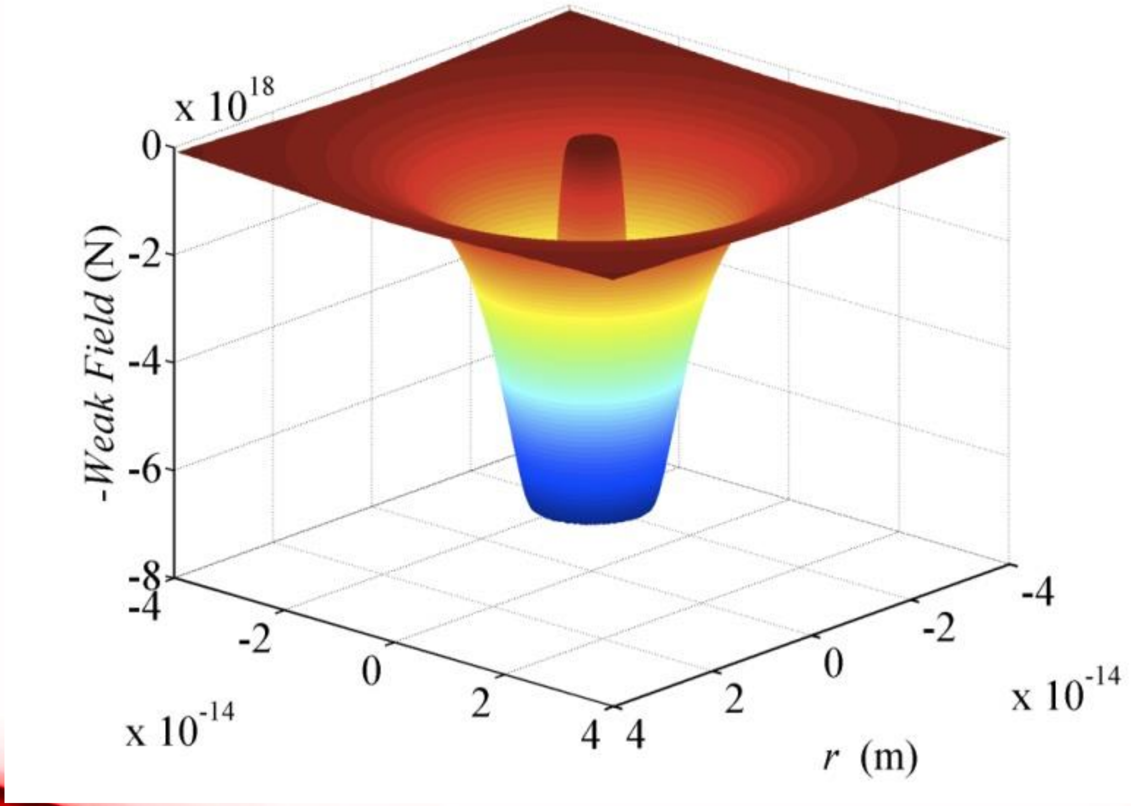
$$F_e(r) = \frac{Q^2}{4\pi\varepsilon_0 r^2} erfc\left(\frac{c_3 \varepsilon_0}{64\pi^2 c_2 \sqrt{2} r}\right)$$





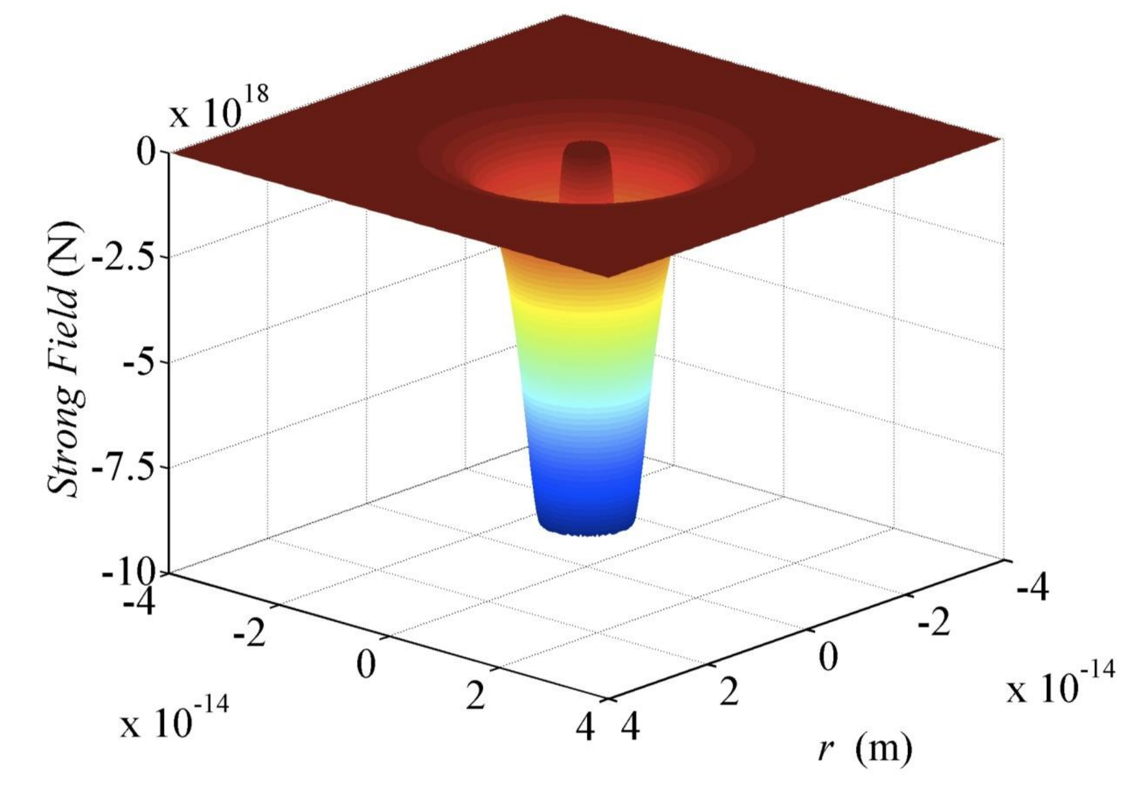
Emergence of a Weak Nuclear Field

$$F_w(r) = \frac{c_3 Q^2}{128 c_2 \pi^3 \sqrt{2\pi r^3}} \exp\left(-\frac{c_3^2 \varepsilon_0{}^2}{2(64\pi^2)^2 c_2^2 r^2} \right)$$



Emergence of a Strong Nuclear Field

$$F_s(r) = -\frac{c_3^3 Q^2 {\varepsilon_0}^2}{24 c_2^3 \pi^3 \sqrt{2\pi}(16\pi)^4 r^5} \exp\left(-\frac{c_3^2 {\varepsilon_0}^2}{2(64\pi^2)^2 c_2^2 r^2}\right)$$



There is nothing such as a free lunch…

There is much more than a free lunch!

Predicting the values of the fundamental constants from various mappings

$$G = \frac{2Ku^2}{c\delta\tau^5}$$

$$\hbar = \frac{c_1}{9\sigma^3\sqrt{N_a}u}$$

$$\frac{\varepsilon_0}{32\pi^2 c_2} = \frac{9GM\hbar\sqrt{\pi}}{8c_1}$$

$$2\sigma_{warp} = \frac{c_3\varepsilon_0}{32c_2\pi^2} = \frac{u_{warp}}{2\pi}$$

$$Q^2 = \frac{(\Delta\mathrm{amu})c^2 c_3\varepsilon_0^2}{18c_2\pi^3}$$

Predicting the values of the fundamental constants from various mappings

$$k = \frac{300\rho_i}{2N_a m_{H_2O}\ 1\ \mathrm{dof/m^3}}\left[\frac{1\ \mathrm{J}}{1\ \mathrm{K}}\right]$$

$$m_e = 0.1 m_{H_2O}\frac{1kg}{M_{Sun}\exp\left[-1/16\pi^2\right]}$$

$$m_p = 0.1\kappa_{\min\max} m_{ref\,\min} = \frac{0.01\ \mathrm{kg}^2}{M_{Earth}}$$

$$N_a \cong \frac{M_{Earth}}{10\exp\left[-1/16\pi^2\right]\ \mathrm{kg}}$$

TOPICS

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I adore the Central Limit Theorem!

- The Central Limit Theorem is an intrinsic property of the convolution of a large number of positive definite functions.
- It can be used the describe a Gaussian star in a flat space.
- The convolution of a Gaussian is a Gaussian...

An Ultimate Generalization!

- The convolution of a large number of Gaussian Stars will converges toward a Gaussian galaxy in a flat space.
- The convolution of a large number of Gaussian galaxies will converges toward a Gaussian Universe...in a flat space.
- Using the same previous coordinate mapping, all these bodies can be described by Curved Gaussians in a curved space!

Take home messages

- According to the present paradigm, the four physical interactions are not empirical laws. They can be seen as emerging phenomena when pattern recognition techniques are used to describe a space-time manifold.
- The fundamental constants of nature can be linked to the unique intrinsic and emergent constant of the model σ and their values can be predicted.

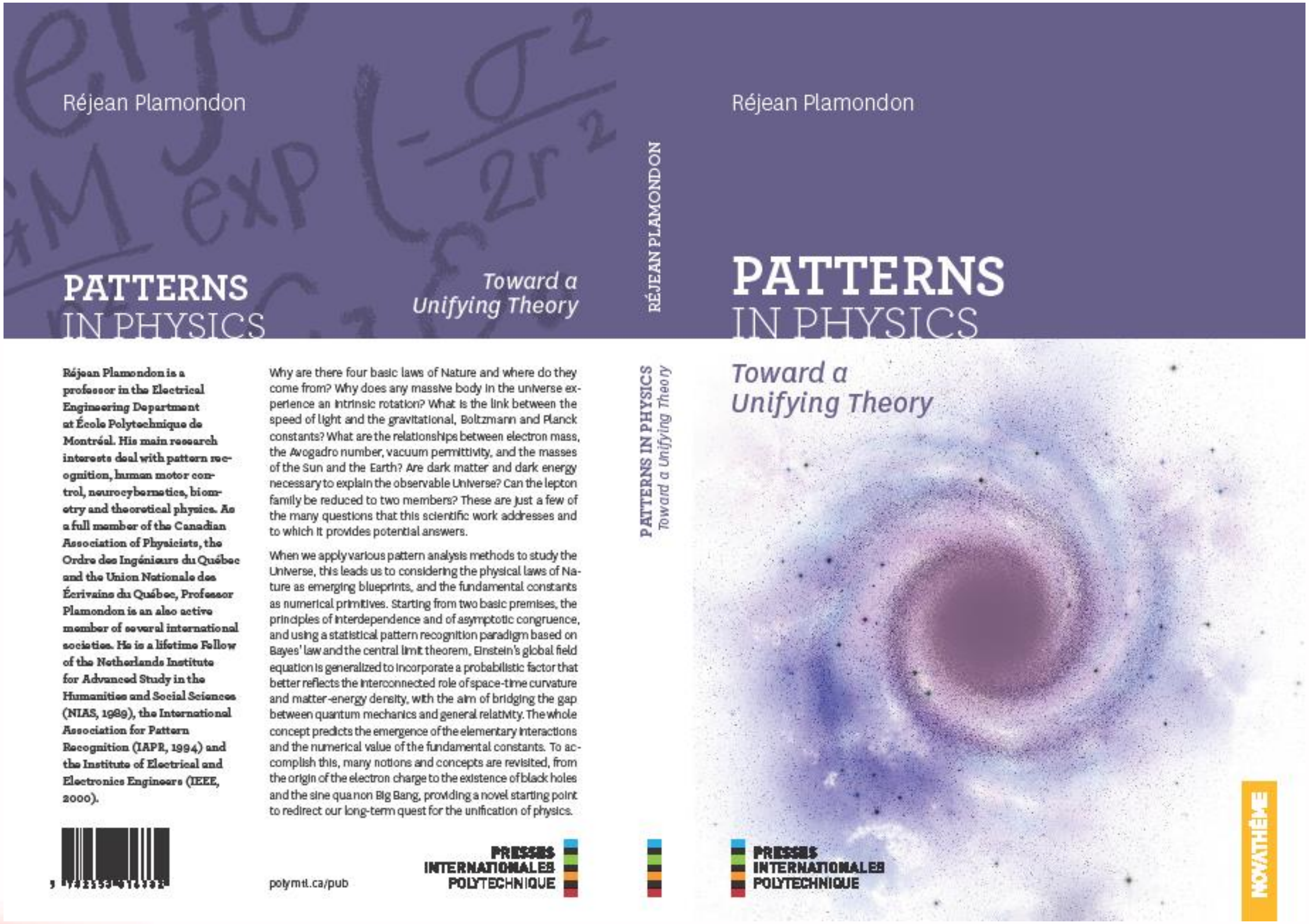
Take home messages

- The model can be applied to some stars, some galaxies and…

  to the whole Universe.

- The Central Limit Theorem is one of the basic tool to study the Signature of the Universe.

To investigate further…



http://www.presses-polytechnique.ca/en/patterns-in-physics

Absolute Relativity

Any body
that beats
in the rhythm
of happiness
holds back
the flight
of his hours

Relativité Absolue.
translated by
David Solvay.

Relativité Absolue

Tout corps

qui bat

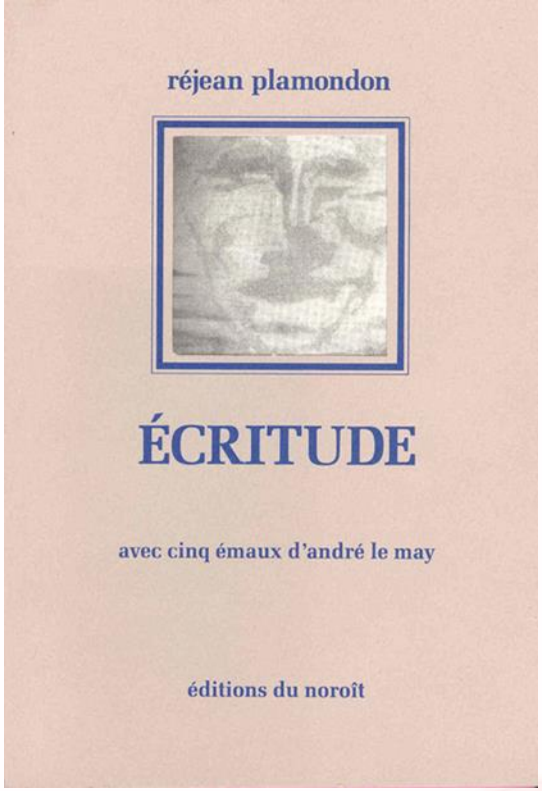
au rythme

du bonheur

retarde

la fuite

de ses heures



Some Intuitive feelings…

PATTERN RECOGNITION MIGHT BE ONE OF THE MOST FUNDAMENTAL SCIENCE!

SOME DOCUMENT ANALYSIS TECHNIQUES

CAN BE APPLIED TO STUDY THE BOOK OF NATURE

(THE CHAPTER ON HUMAN BEINGS)

AS WELL AS

THE ENCYCLOPEDIA OF THE UNIVERSE!

THE STUDY OF THE UNIVERSE IS A VERY THOUGH SIGNATURE ANALYSIS PROBLEM

ONE SIGNER…
ONE SIGNATURE SAMPLE…

Where do I go From here ?

Back to Poetry

and

to my Ultimate Long Term Quest

Two

Two signatures
engraved
on the back
of the Milky
Way

a promise
of fidelity
to fleeting
embraces

Deux,
translated
by Andrea Zanin.

Two

Two signatures
engraved
on the back
of the Milky
Way

a promise
of fidelity
to fleeting
embraces

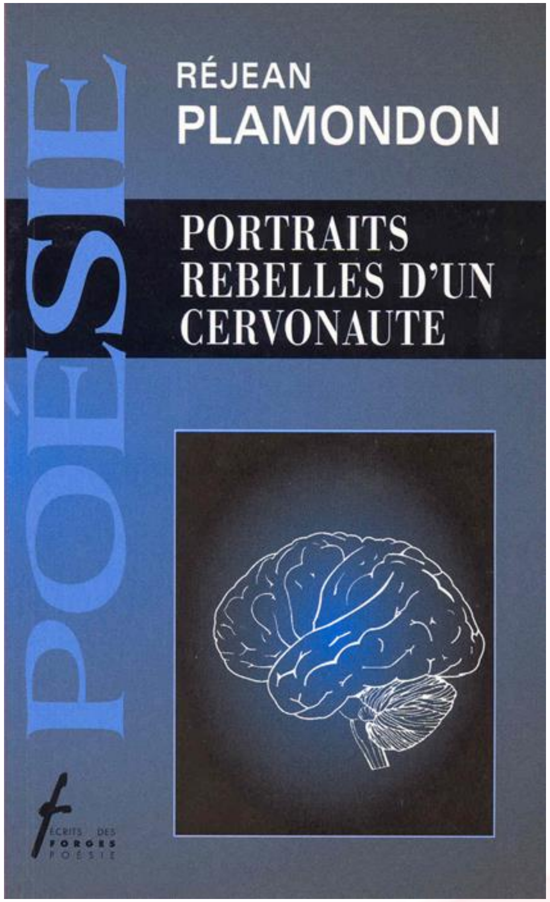
Deux,
translated
by Andrea Zanin.

•

Deux

Deux signatures
gravées
au verso
de la voie
lactée

en gage
de fidélité
aux étreintes
filantes



A Search for the Missing Links ...

Between Two Emergent Patterns: Lognormality and Curved Gaussianity…

Mute

Ink coagulates
in my veins

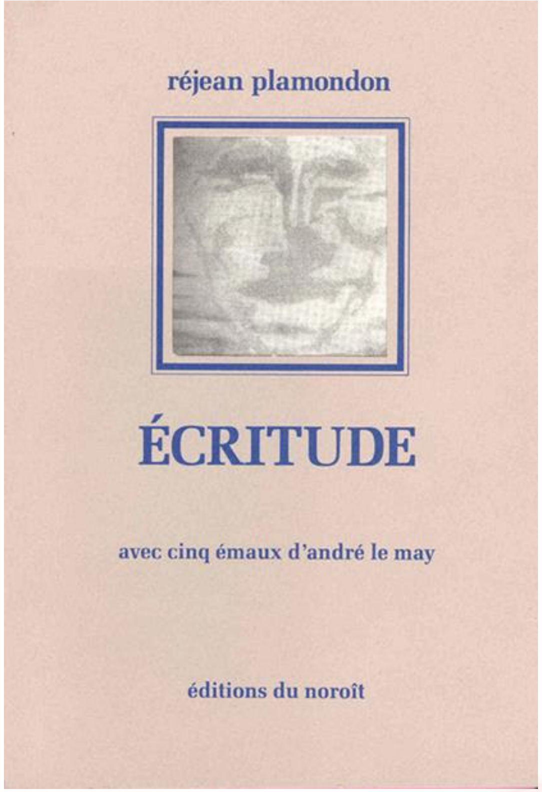
My hands clumsy
against rough paper
that vainly seeks
a cursive meaning
in my faltering attempts

Mutisme,
translated by
Andrea Zanin

Mutisme

L'encre se coagule
dans mes veines

Mes mains boitent
sur du papier rêche
qui cherche en vain
un sens cursif
à mes balbutiements



The Quest for Lognormality in a Curved Gaussian Space-Time: An On-line Handwriting Generation Journey.

QUESTIONS ?